

# Post-Quantum Cryptography

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**CROSSING**

<http://www.crossing.tu-darmstadt.de>

# Outline



- What is public-key cryptography? PK encryption, signatures
- Cybersecurity requires PKC
- Current PKC
  - RSA
  - Hardness of factoring
  - Hardness of DL
- Quantum computer threat
- Post quantum strategy

# Outline



- Multivariate PKC
- Code-based PKC
- The idea of lattice-based PKC
- State-of-the art lattice-based signatures
  - overview
  - LWE: simplify explanation?
  - SIS: simplify explanation?
- Tesla
  - Tesla: improve explanation, compare with RSA with parameters shown earlier

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# Outline



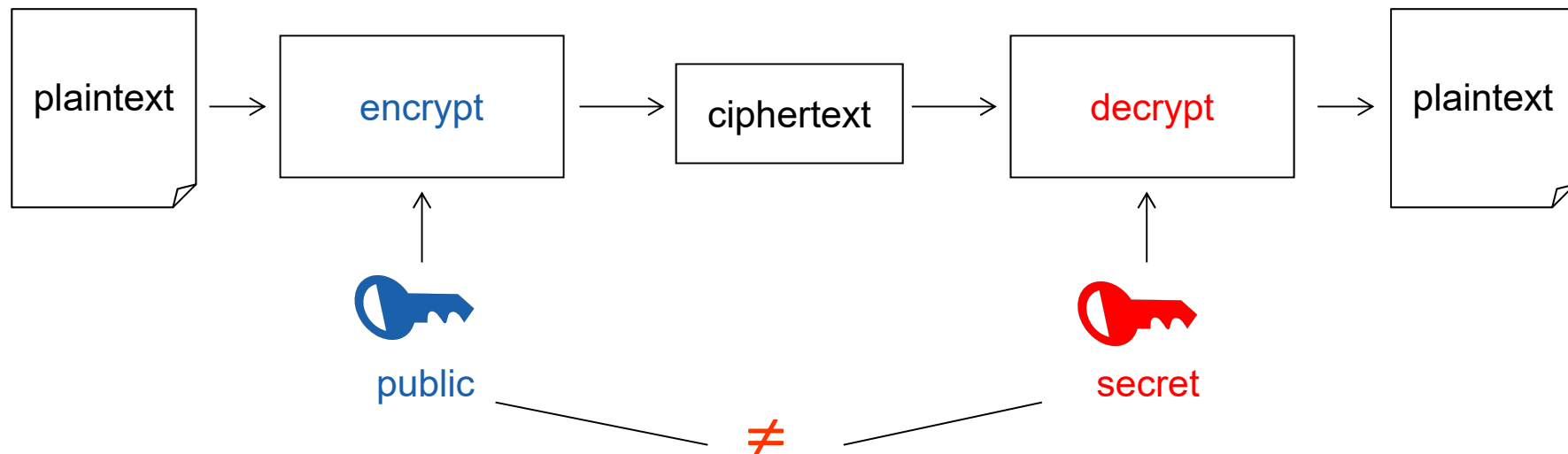
- State-of-the art lattice-based PK encryption
  - overview
- Lara
  - Improve explanation, compare with RSA with parameters shown earlier
- Hash-based signatures
  - The paradigm
  - Security
  - How it works: please add more
  - Performance
- Conclusion



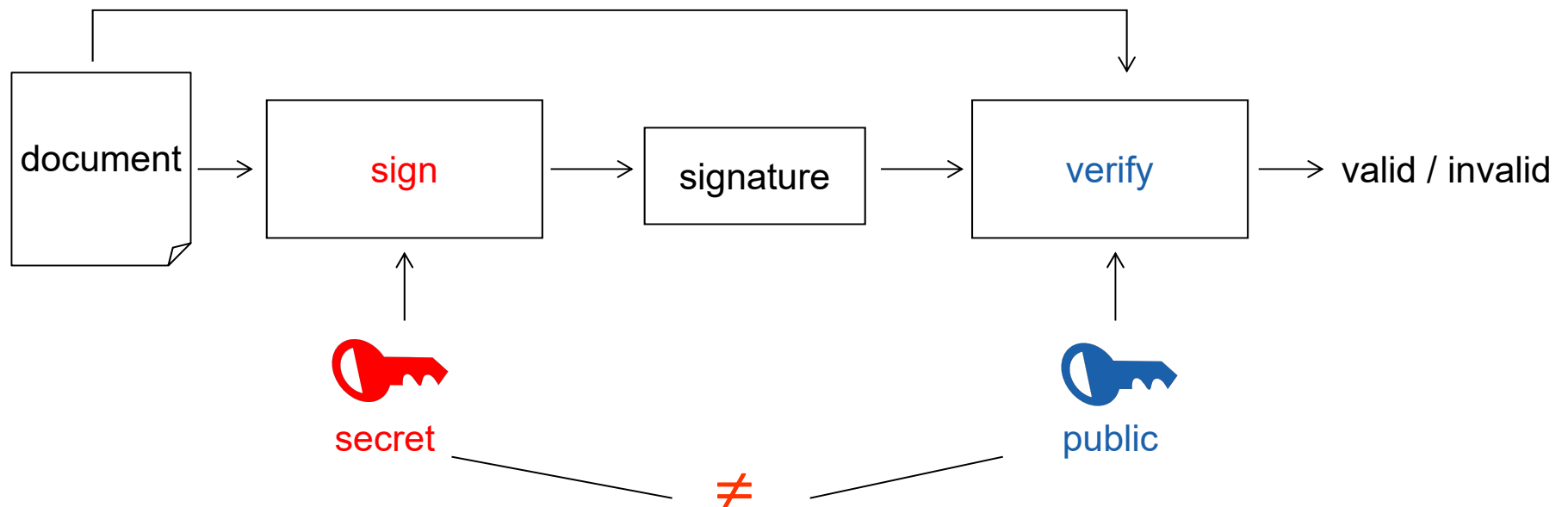
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# Public-key cryptography

# Public-key encryption



# Digital signatures

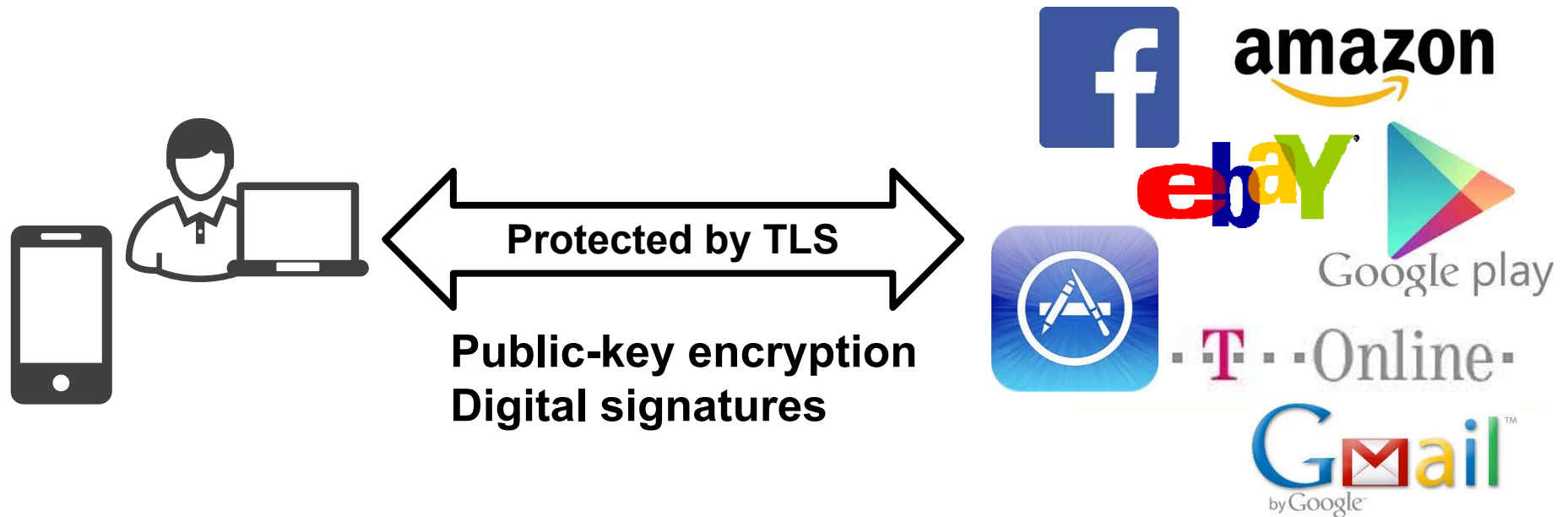




# IT-security requires public-key cryptography



# Communication on the Internet



**Billions daily**  
**e.g., 1.23 billion Facebook log ons daily**

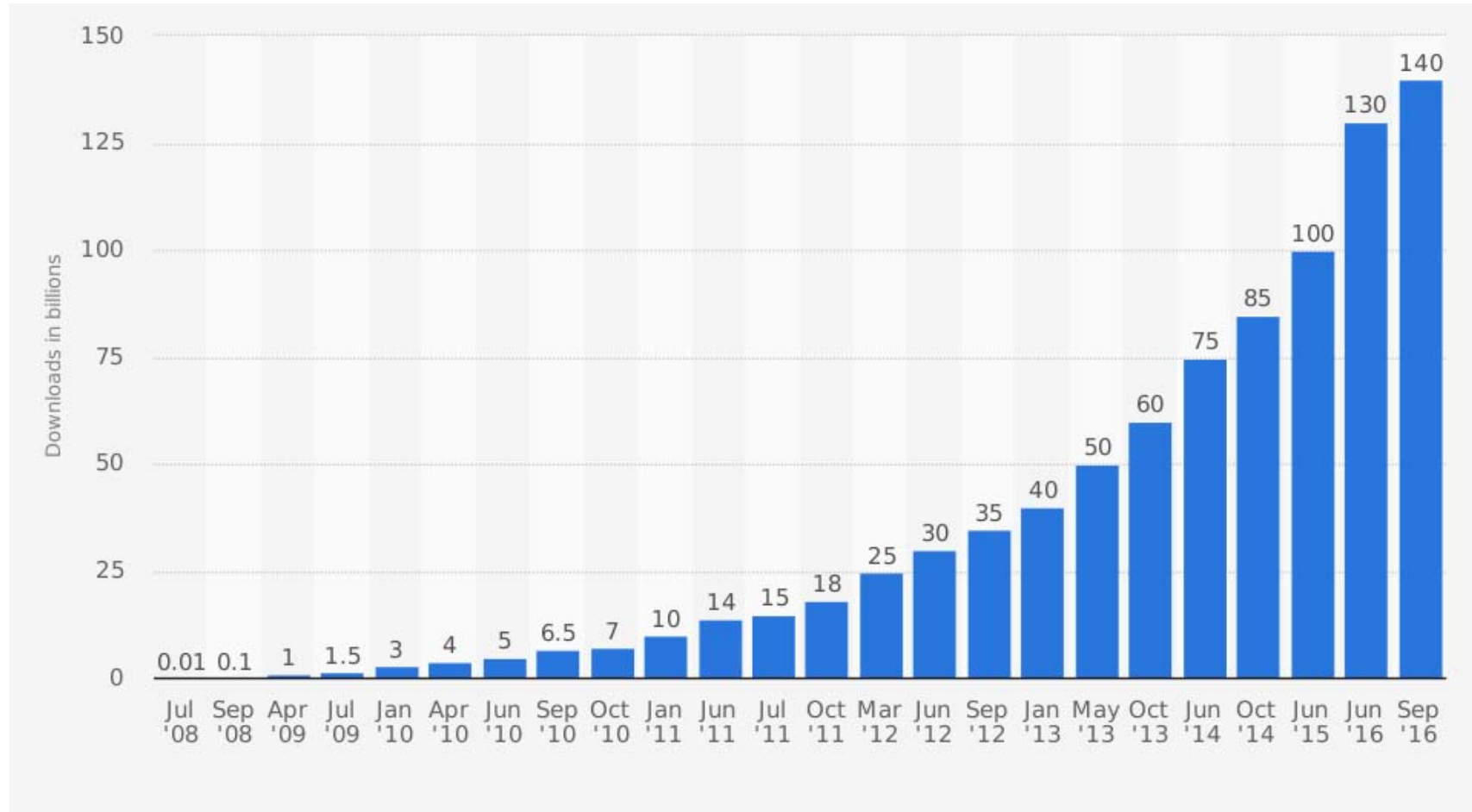
# Software downloads



# Cumulative number of apps downloaded from the Apple App Store from July 2008 to September 2016 (in billions)



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Source: Apple, TechCrunch; © Statista 2017



# Current public-key cryptography

# “Generic” RSA



Public key: finite Group  $G$ , exponent  $e$ ,  $\gcd(e, |G|) = 1$

Secret key:  $|G|$

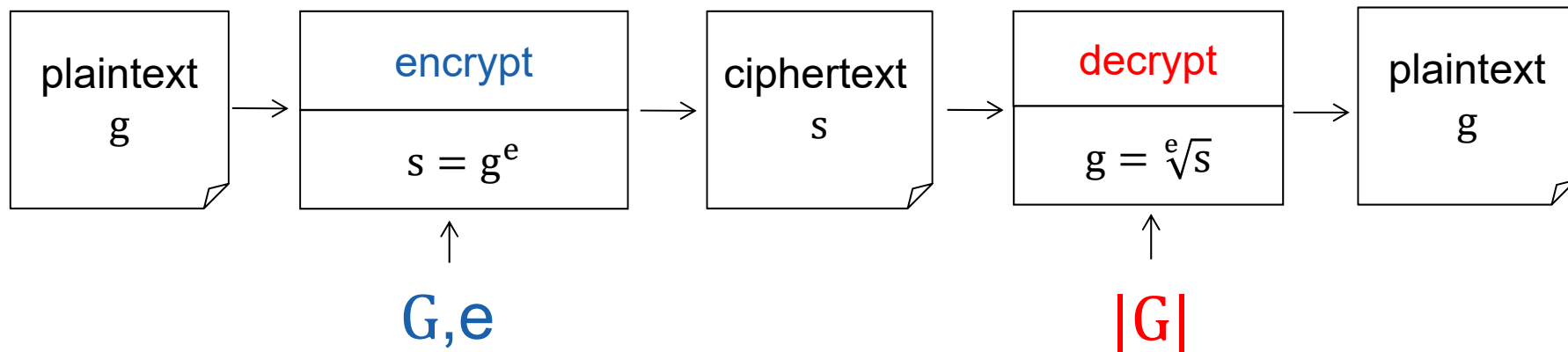
Allows to compute:  $\sqrt[e]{g} = g^{e^{-1} \bmod |G|}, g \in G$

# “Generic” RSA encryption

Public key: finite Group  $G$ , exponent  $e$ ,  $\gcd(e, |G|) = 1$

Secret key:  $|G|$

Allows to compute:  $e\sqrt{g} = g^{e^{-1} \bmod |G|}, g \in G$



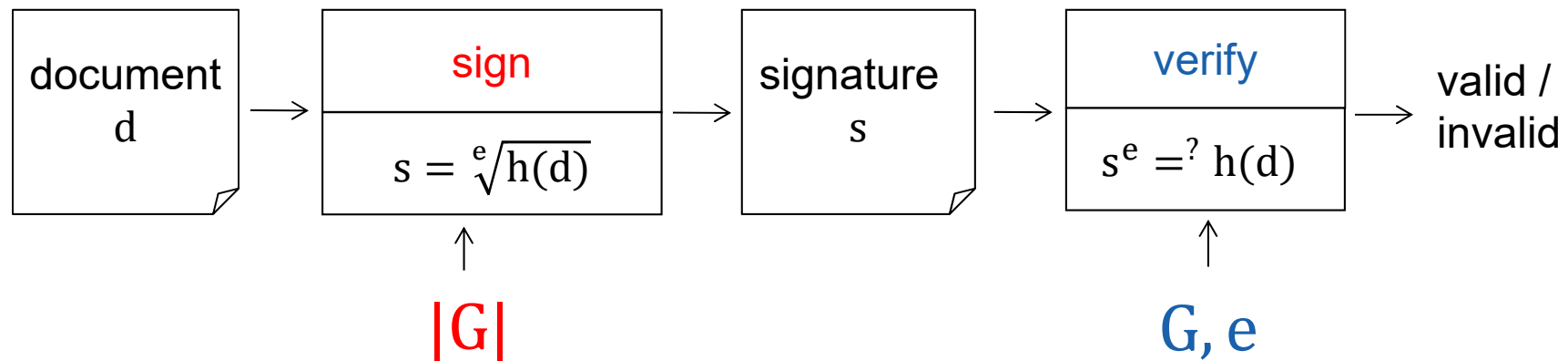
# “Generic” RSA signature

Public key: finite Group  $G$ , exponent  $e$ ,  $\gcd(e, |G|) = 1$

Secret key:  $|G|$

Allows to compute:  $e\sqrt{g} = g^{e^{-1} \bmod |G|}, g \in G$

Hash function  $h: \{0,1\}^* \rightarrow G$



# RSA: How to keep $|G|$ secret?



Set up:  $p, q$  primes,  $n = pq$ ,  $G = (\mathbb{Z}/n\mathbb{Z})^*$ ,  $e$ ,  $\gcd(e, |G|) = 1$

$$|G| = (p - 1)(q - 1),$$

Public key:  $(n, e)$

Secret key:  $d = e^{-1} \bmod (p - 1)(q - 1)$

Security relies on hardness of integer factorization

Only known method to keep  $|G|$  secret



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# Public RSA key of *PayPal*

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31795268810366627125473790859797098391197908286525435077364011285  
90510438382632507968446475666470736765037698350040734989120408350  
36111984436982786965149879673665411793622083038631384645332380078  
74977706229020370398442691648609936395220964249973923183224282326  
24293883124379061631765073423204610042801378799675461282344132598  
82008909669991881742777224061960485068828406517329900151157317659  
33488273881059259173651847367586007177868818486949631199170802343  
43393438632241104852580095512302299147769809327477605192706038053  
13338263751205344637414772085776930403119514835209366439467587236  
52946961075123119618309889468210461323294360350311459316891189249

# ElGamal encryption and signatures



Rely on the **Discrete Logarithm Problem**:

Given: Group  $G = \langle g \rangle$ ,  $h \in G$

Find:  $x \in \mathbb{N}$  with  $h = g^x$

Choices for  $G$ :  $\text{GF}(p^n)^*$

Group of points of elliptic curves over  $\text{GF}(p^n)$



# How difficult is factoring and DL?

# Shor's algorithm 1997



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## Polynomial-Time Algorithms for Prime Factorization and Discrete Logarithms on a Quantum Computer\*

Peter W. Shor<sup>†</sup>



**RSA and ElGamal  
insecure**

A digital computer is generally believed to be a device; that is, it is believed that an increase in computation time leads to an increase in the number of digits of the integer to be factored. This paper considers the possibility that this is not true when quantum mechanics is taken into consideration. This paper considers factoring integers and finding discrete logarithms, two problems which are generally thought to be hard on a classical computer and which have been used as the basis of several proposed cryptosystems. Efficient randomized algorithms are given for these two problems on a hypothetical quantum computer. These algorithms take a number of steps polynomial in the input size, e.g., the number of digits of the integer to be factored.

**Keywords:** algorithmic number theory, prime factorization, discrete logarithms, Church's thesis, quantum computers, foundations of quantum mechanics, spin systems, Fourier transforms

**AMS subject classifications:** 81P10, 11Y05, 68Q10, 03D10

# Quantum computer realistic



www.datacenterdynamics.com/content-tracks/servers-storage/google-may-unveil-a-powerful-quantum-computer-by-end-of-2017/96880.fullarticle 133%

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# Google may unveil a powerful quantum computer by end of 2017

2 September 2016 | By **Sebastian Moss**



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# Post-quantum cryptography

# Performance requirements

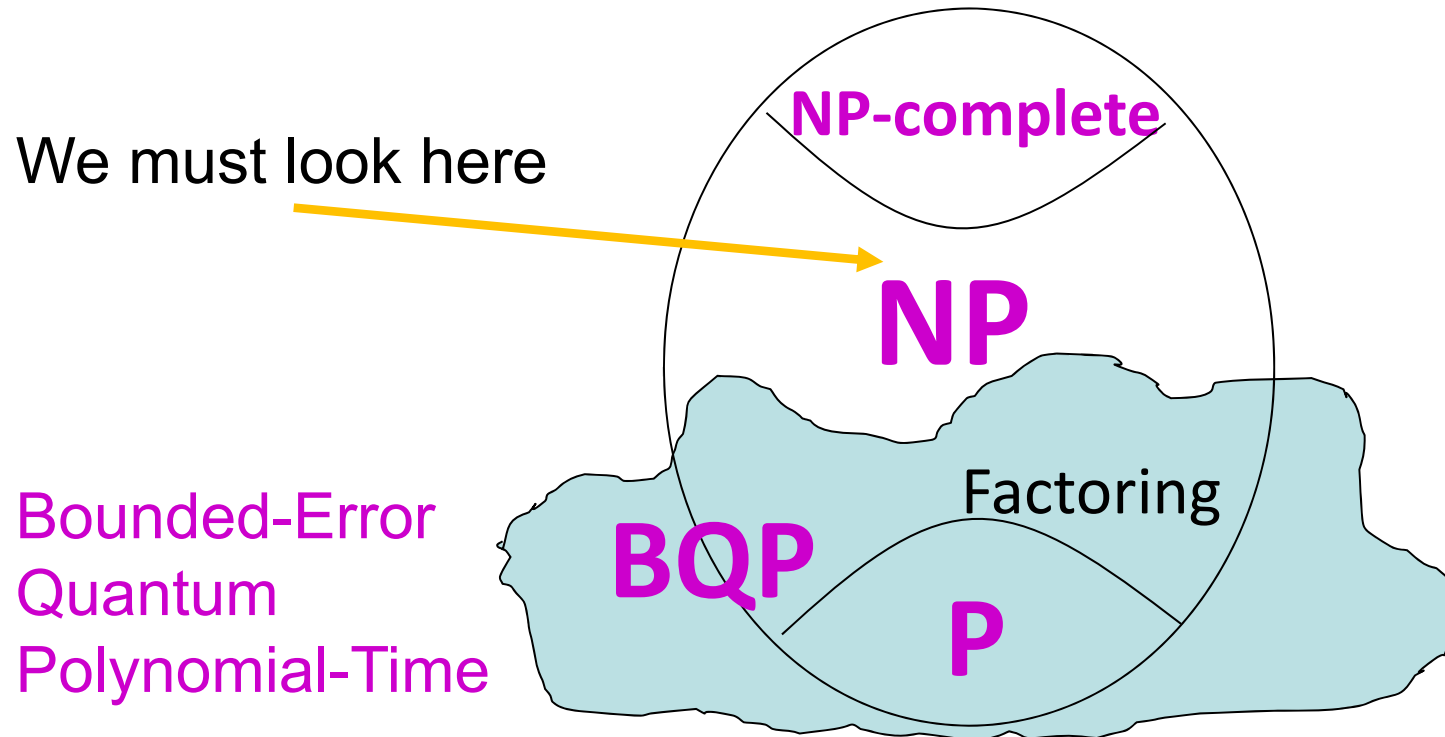
Secure from-until	Security level	RSA modulus/finite field size	Elliptic curve
2017-2020	96	1776	192
2017-2030	112	2432	224
2017-2040	128	3248	256
2017- ?	256	15424	512

## Ecrypt recommendations

- Space for keys and signatures: a few kilobytes
- Times: milliseconds

# Post-quantum problems?

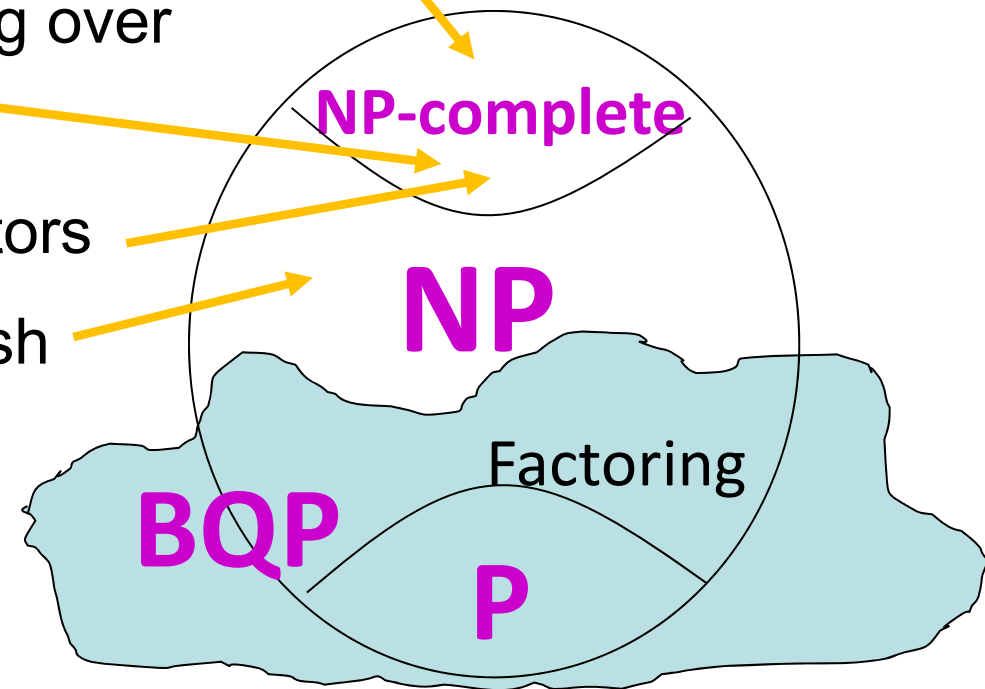
No provably quantum resistant problems





# Candidates

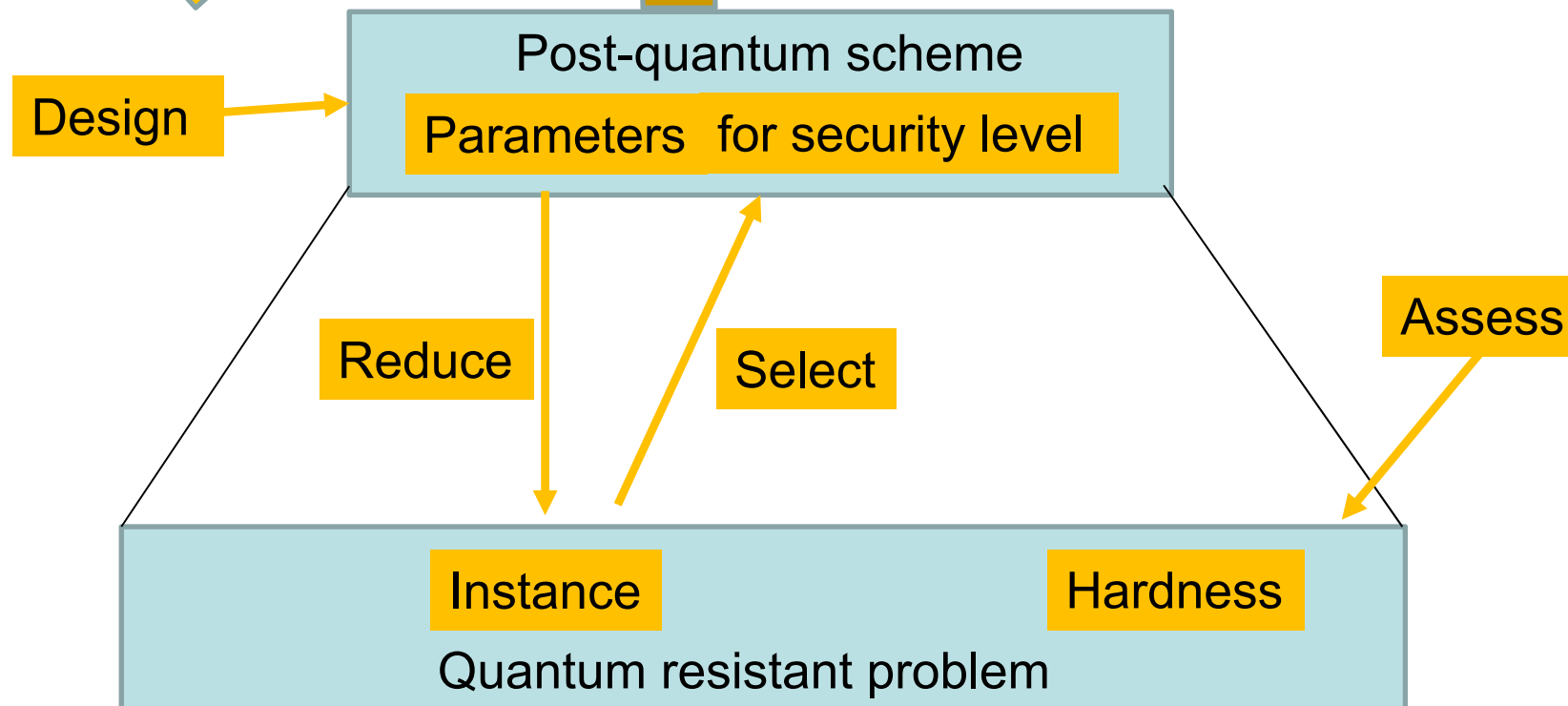
- Solving non-linear equation systems over finite fields
- Bounded distance decoding over finite fields
- Short and close lattice vectors
- Breaking cryptographic hash functions



# Strategy



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# Strategy - Methodology

Task	Goal	Method
Design	Efficient crypto with (tight )reduction proof to hard algorithmic problem	Algorithmics, quantum complexity theory
Assess	Quantum resistant problems, quantum time-space complexity, worst-to-average-case reduction	Quantum algorithmics, quantum complexity theory, parallel computing
Select	Parameter sets for given security level	Explicit complexity analysis



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# Multivariate cryptography

# MQ problem

$$4x + x^2 + y^2z \equiv 1 \pmod{13}$$

$$7y^2 + 2xz^2 \equiv 12 \pmod{13}$$

$$x + y^2 + 12xz^2 \equiv 4 \pmod{13}$$

Solution:  $x = 15$ ,  $y = 29$ ,  $z = 45$

# MQ-Problem



Given:  $n, m, p_1, \dots, p_m \in F[x_1, \dots, x_n]$  quadratic,  $F$  finite field

Find:  $y_1, \dots, y_n \in F$ , such that

$$p_1(y_1, \dots, y_n) = \dots = p_m(y_1, \dots, y_n) = 0$$

MQ is NP-complete (Garey, Johnson 1979) (decision version)

# MQ Challenge



## Fukuoka MQ Challenge

### News

**2016/12/17** Type I of  $n=74$  and  $m=148$  was solved by Antoine Joux.

**2016/12/17** Type I of  $n=73$  and  $m=146$  was solved by Antoine Joux.

**2016/12/13** Type I of  $n=72$  and  $m=144$  was solved by Antoine Joux.

**2016/12/13** Type I of  $n=71$  and  $m=142$  was solved by Antoine Joux.

**2016/12/13** Type I of  $n=70$  and  $m=140$  was solved by Antoine Joux.

**2016/12/13** Type I of  $n=69$  and  $m=138$  was solved by Antoine Joux.

[more>>](#)

### Introduction

Welcome to the Fukuoka MQ challenge project.

Multivariate Quadratic polynomial (MQ) problem is the basis of security for potentially

[Submission](#)

[Guide for Participants](#)

- [How to participate](#)
- [Challenge Format](#)

[Download Challenges](#)

**Encryption**  
( $m=2n$ )

[Type I](#) [Type II](#) [Type III](#)

Toy examples and answers  
of  $n=10, 15, 20$

[10](#) [15](#) [20](#)

# Multivariate signatures



$P: F^n \rightarrow F^m$ , easily invertible non-linear

$S: F^n \rightarrow F^n$ ,  $T: F^m \rightarrow F^m$ , affine linear

Public key:  $G = S \circ P \circ T$ , hard to invert

Secret Key:  $S, P, T$  allows to find  $G^{-1}$

$$G^{-1} = T^{-1} \circ P^{-1} \circ S^{-1}$$

Signing:  $s = T^{-1} \circ P^{-1} \circ S^{-1}(m)$

Verifying:  $G(s) \stackrel{?}{=} m$

- UOV, Goubin et al., 1999
- Rainbow, Ding, et al. 2005
- pFlash, Cheng, 2007
- Gui, Ding, Petzoldt, 2015
- QUARTZ, Patarin, Courtois, 2001

Forging signature: Solve  $G(s) - m = 0$



# Performance of multivariate signature schemes (80-bit security)

Scheme	Cyclecounts [k-cycles]		Sizes		
	Sign	Verify	pk [kB]	sk [kB]	sig. [bit]
<b>Gui-96</b>	596	70	62	3	126
<b>Gui-95</b>	1,441	60	30	3	120
<b>Rainbow</b>	4,740	350	25	19	344
<b>UOV</b>	11,201	230	14	96	672
<b>QUARTZ</b>	315,716	84	72	3	128
<b>RSA-1024</b>	1,058	74	0.125	0.125	1024
<b>ECDSA P160</b>	558	635	0.039	0.059	320

# Hardware implementations of multivariate signature schemes



First approaches:

- Rainbow:  
*Fast Multivariate Signature Generation in Hardware: The Case of Rainbow*; Balasubramanian, Bogdanov, Rupp, Ding, Carter  
2008
- UOV:  
*High-Speed Hardware Implementation of Rainbow Signature on FPGAs*; Tang, Yi, Ding, Chen, Chen  
2011
- Gui: no approaches yet



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# Code-based cryptography

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# Bounded distance decoding problem

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- Given:
- Linear code  $C \subseteq \mathbb{F}_2^n$
  - $y \in \mathbb{F}_2^n$
  - $t \in \mathbb{N}$

- Find:
- $x \in C: \text{dist}(x, y) \leq t$

BDD is NP-complete (Berlekamp et al. 1978) (Decisional version)

# McEliece cryptosystem (1978)

$S, G, P$  matrices over  $F$

$G$  generator matrix for Goppa code



Allows to  
solve BDD

Public key:  $G' = S \circ G \circ P, t$

Secret Key:  $P, S, G$

Encryption:  $c = mG' + z \in F^n$

Decryption:  $x = cP^{-1} = mSG + zP^{-1}$

solve BDD to get  $y = mSG$

decode to obtain  $m$

# Performance of code-based encryption schemes (80-bit security)



Scheme	Cyclecounts [k-cycles]		Size		Expansion factor
	Encryption	Decryption	pk [kB]	sk [kB]	
McEliece QC-MDPC [vMG14]	7,018	42,130	0.586	0.180	2
RSA-1024 [GPWES04]	3,440	87,920	0.125	0.125	1
ECC-ecp160r1 [GPWES04]	6,480	6,480	0.039	0.059	1

**[vMG14]:** von Maurich and Güneysu: Towards side-channel resistant implementations of QC-MDPC McEliece encryption on constrained devices, PQCrypto 2014

**[GPWES04]:** Gura, Patel, Wander, Eberle, Shantz: Comparing elliptic curve cryptography and RSA on 8-bit cpus, CHES 2004



# Lattice-based cryptography

# Why lattice-based cryptography?



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- Expected to resist quantum computer attacks
- Worst-to-average-case reduction
- Permits fully homomorphic encryption and many other applications

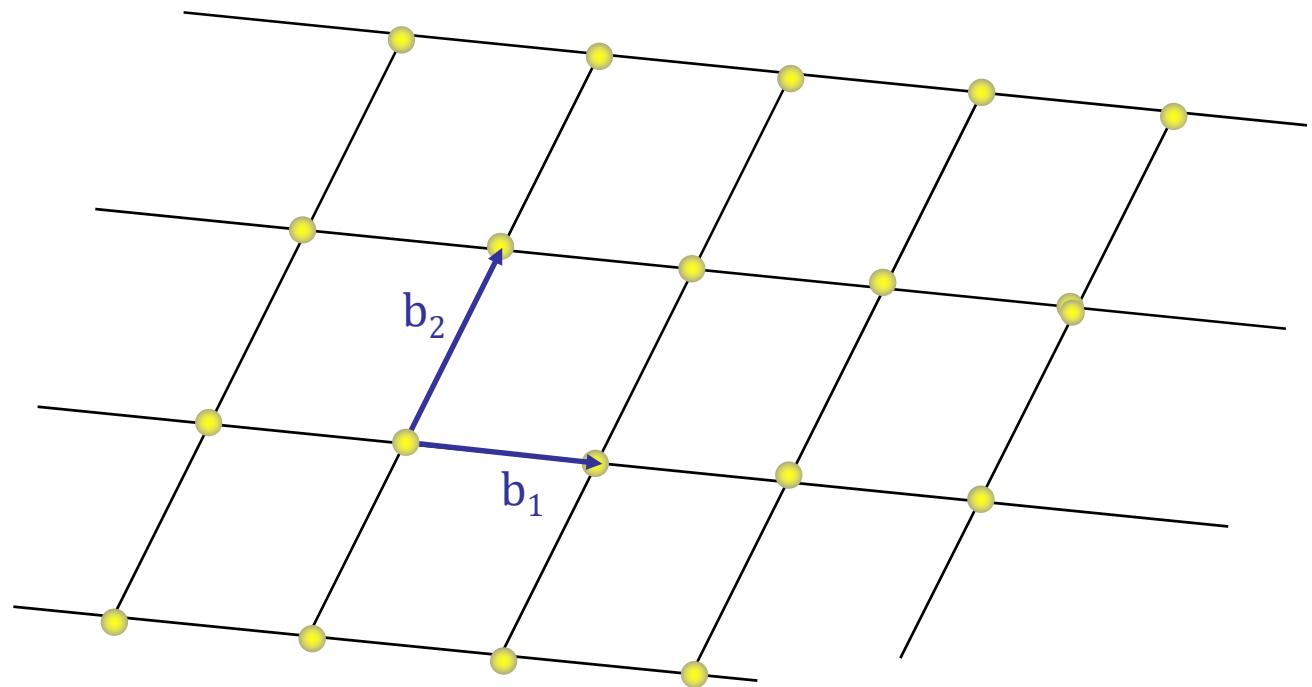




# The idea of lattice-based cryptography

## 2-dimensional lattice

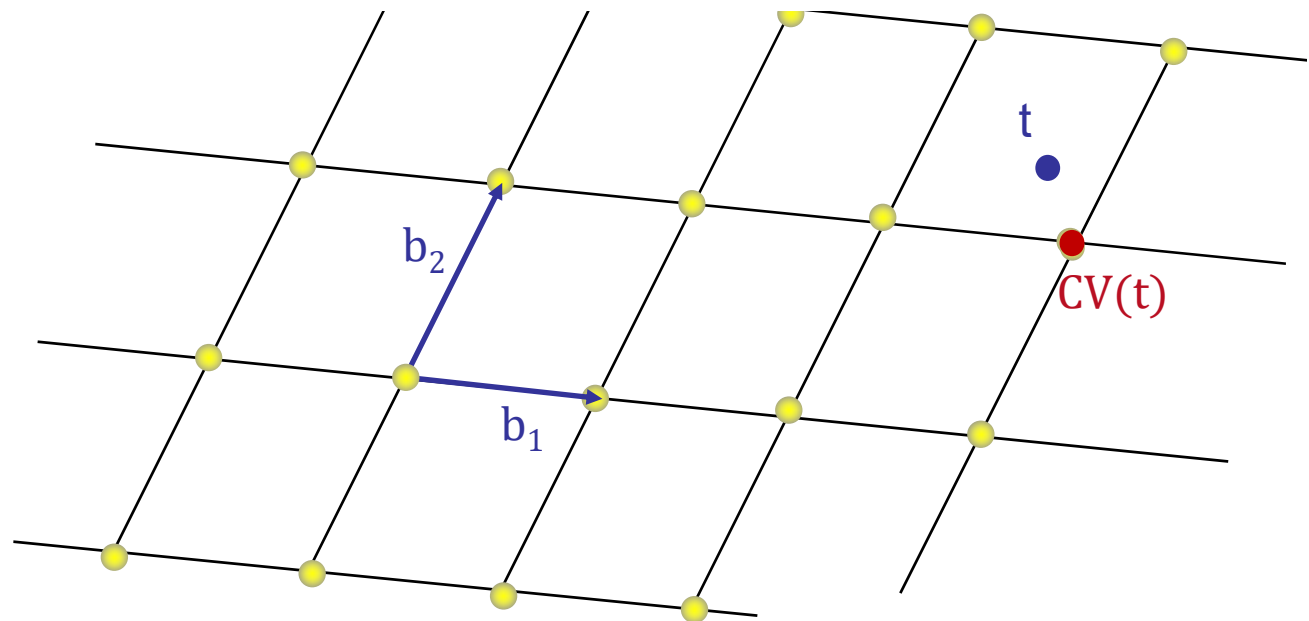
$$B = (b_1, b_2), L(B) = \mathbb{Z}b_1 + \mathbb{Z}b_2$$



## 2-dimensional CVP

Given:  $B = (b_1, b_2)$ ,  $t, \alpha$

Find:  $CV(t) \in L(B): \|t - CV(t)\| \leq \min_{w \in L} \|t - w\|$



# Lattice problems



$n \in \mathbb{N}, L = \mathbb{Z}b_1 + \dots + \mathbb{Z}b_n \subseteq \mathbb{R}^n$  lattice;  $B = (b_1, \dots, b_n)$  basis

## $\alpha$ -Closest Vector Problem (CVP)

Given:  $\alpha > 1$ , lattice  $L = L(B)$  basis  $B$ ,  $t$

Find:  $v \in L$  such that  $\|t - v\| \leq \alpha \min_{w \in L} \|t - w\|$

## $\alpha$ -Shortest Vector Problem (SVP)

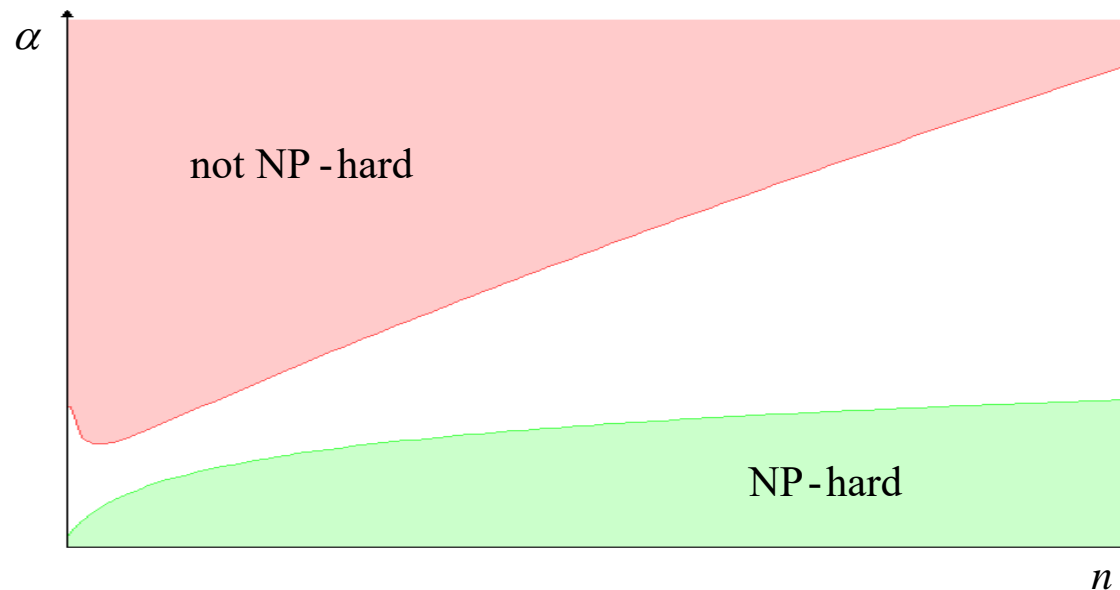
Given:  $\alpha > 1$ , lattice  $L = L(B)$  in terms of basis  $B$

Find:  $v \in L$  nonzero such that  $\|v\| \leq \alpha \lambda_1(L)$

# Complexity of $\alpha$ -CVP/SVP

Arora et al. (1997):

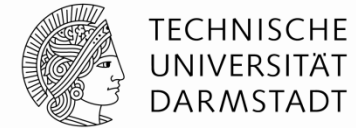
$\log(n)^c$  - CVP/SVP is NP - hard for all  $c$



Goldreich, Goldwasser (2000):

$\Omega(\sqrt{n} / \log(n))$  - CVP/SVP is not NP - hard or  $\mathbf{coNP} \subseteq \mathbf{AM}$

# Practical complexity



## TU DARMSTADT LATTICE CHALLENGE

### INTRODUCTION

Welcome to the lattice challenge.

<https://www.latticechallenge.org/>

does not mean that one can solve all instances simultaneously, but rather that one can solve even the worst case instances. We think these lattice bases are hard instances and most fitting to test and compare modern lattice reduction algorithms.

We show how these lattice bases were constructed and prove the existence of short vectors in each of the corresponding lattices in [2]. We challenge everyone to try whatever means to find a short vector. There are two ways to enter the hall of fame:

### SUBMISSION

Submission

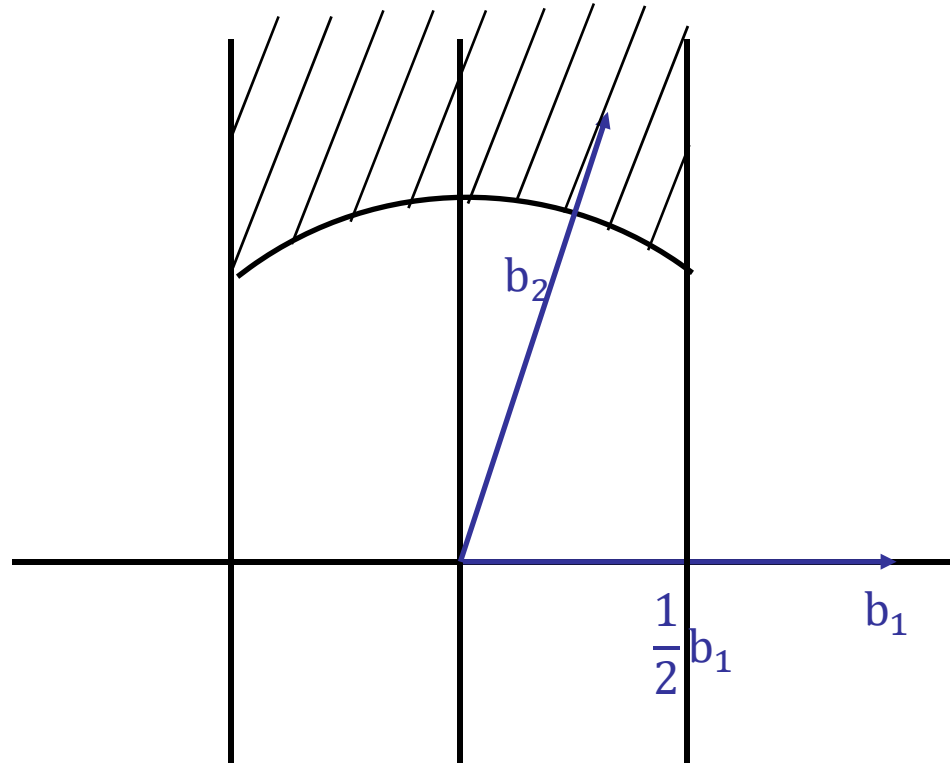
### DOWNLOAD

Format of Challenge Files

Toy Challenges in Dimension

200 225 250 275  
300 325 350 375  
400 425 450 475

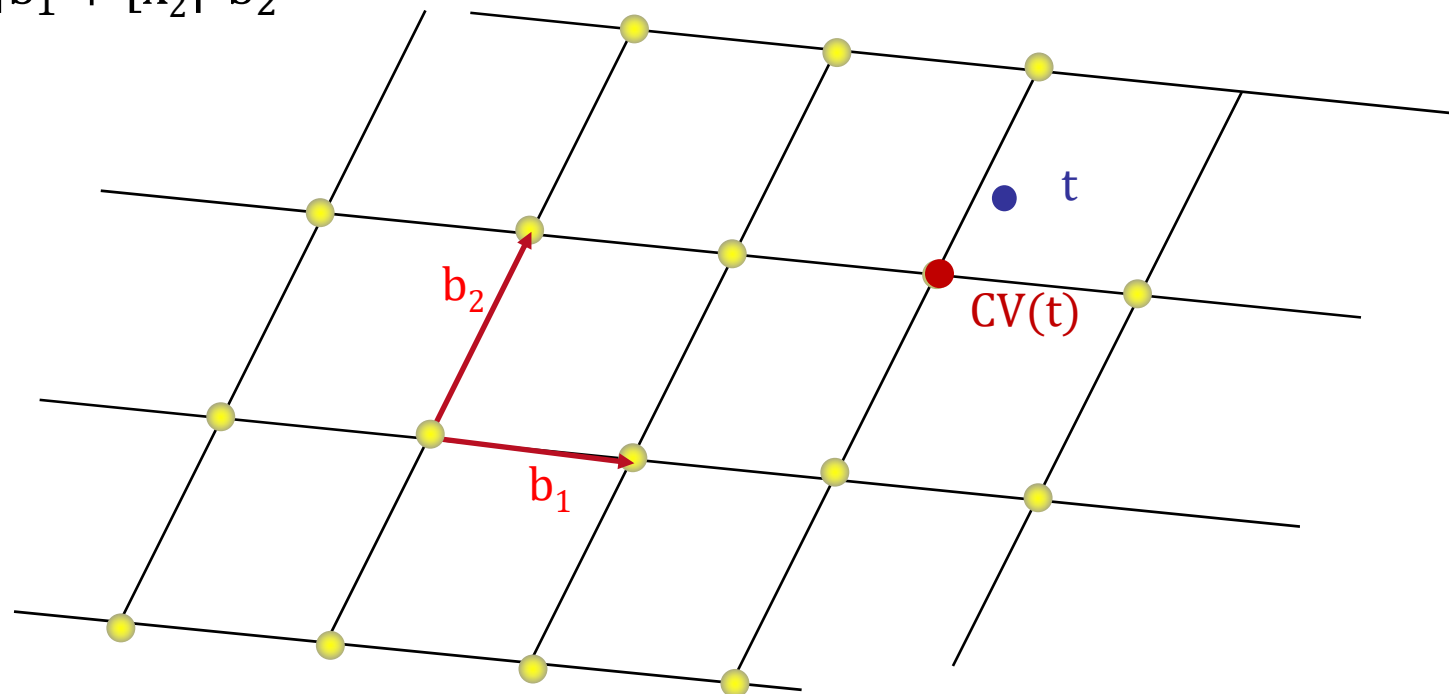
# Reduced bases (Gauß 1801)



# $(b_1, b_2)$ reduced Gauss: CVP easy

$$t = x_1 b_1 + x_2 b_2$$

$$CV(t) = \lfloor x_1 \rfloor b_1 + \lfloor x_2 \rfloor b_2$$





**B = (b<sub>1</sub>, b<sub>2</sub>) not reduced ⇒ CVP hard**



$$L = \mathbb{Z}^2, \mathbf{B} = \left( \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right), \mathbf{t} = \begin{pmatrix} 3.4 \\ -2.3 \end{pmatrix}, \text{CVP}(\mathbf{t}) = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$$

$$\text{Another basis } \mathbf{B}' = \left( \begin{pmatrix} 100 \\ 99 \end{pmatrix}, \begin{pmatrix} 99 \\ 98 \end{pmatrix} \right)$$

$$\mathbf{t} = \begin{pmatrix} 3.4 \\ -2.3 \end{pmatrix} = -560.9 \cdot \begin{pmatrix} 100 \\ 99 \end{pmatrix} + 566.6 \cdot \begin{pmatrix} 99 \\ 98 \end{pmatrix}$$

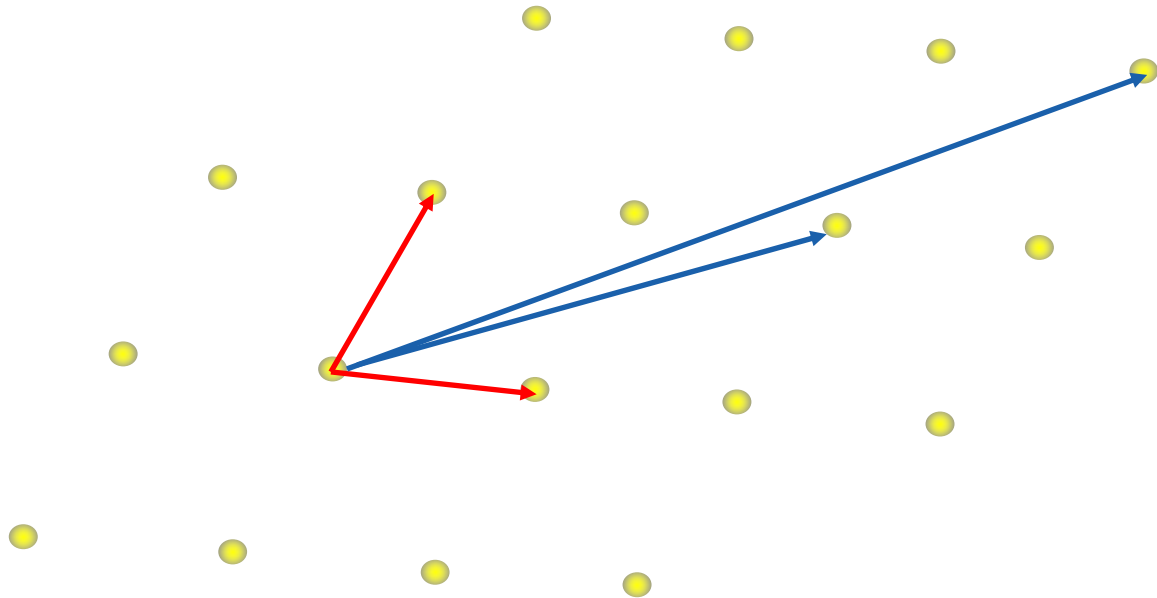
$$-561 \cdot \begin{pmatrix} 100 \\ 99 \end{pmatrix} + 567 \cdot \begin{pmatrix} 99 \\ 98 \end{pmatrix} = \begin{pmatrix} 33 \\ 27 \end{pmatrix} \neq \begin{pmatrix} 3 \\ -2 \end{pmatrix} = \text{CVP}(\mathbf{t})$$

# Key generation

Key generation:  $n \in \mathbb{N}$ ,  $L \subseteq \mathbb{R}^n$  lattice

**Secret key:** „reduced“ basis  $B$  of  $L$ . (Allows to efficiently solve CVP.)

**Public key:** „bad“ basis  $B'$  of  $L$ . (Does not.)



# Public-key encryption

Plaintext  $v \in L$

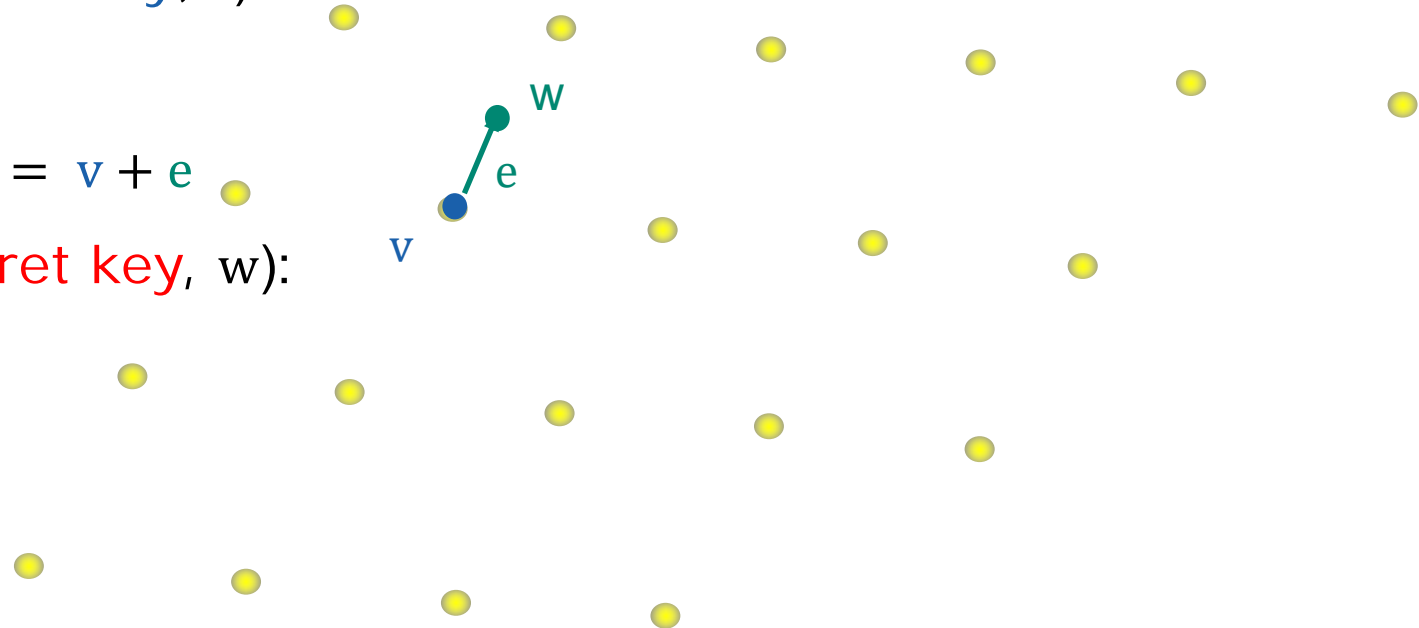
Encryption(public key,  $v$ )

- small  $e \in \mathbb{R}^n$

- ciphertext  $w = v + e$

Decryption(secret key,  $w$ ):

-  $v = CV(w)$



# Digital signature

Public: Cryptographic hash function  $h: \{0,1\} \rightarrow \mathbb{R}^n$

Sign(**secret key**, document  $d$ ):

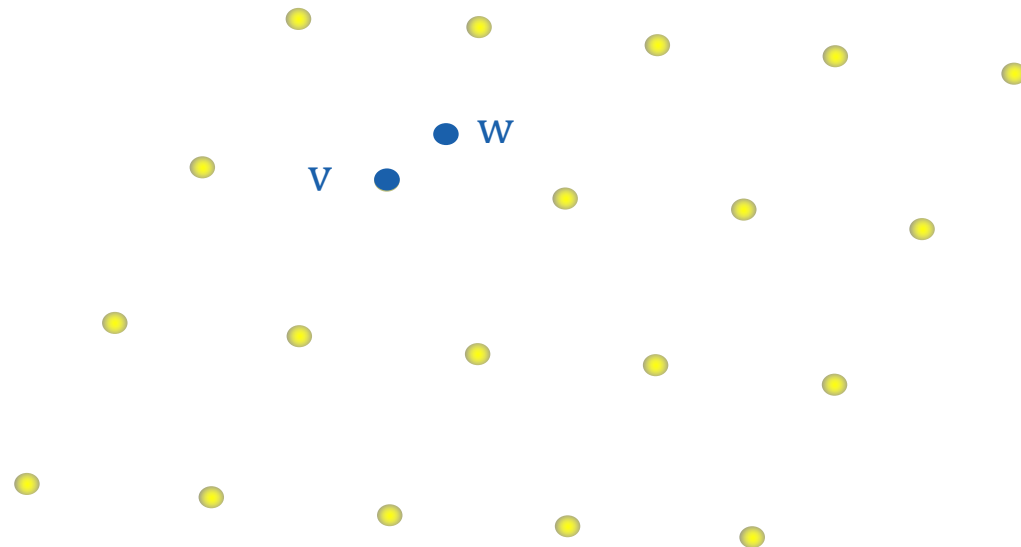
$$w = h(d)$$

$$v = CV(w)$$

Verify(**public key**,  $v, w$ ):

$v$  lattice vector?

$v$  close to  $w$  ?



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## Early lattice-based schemes

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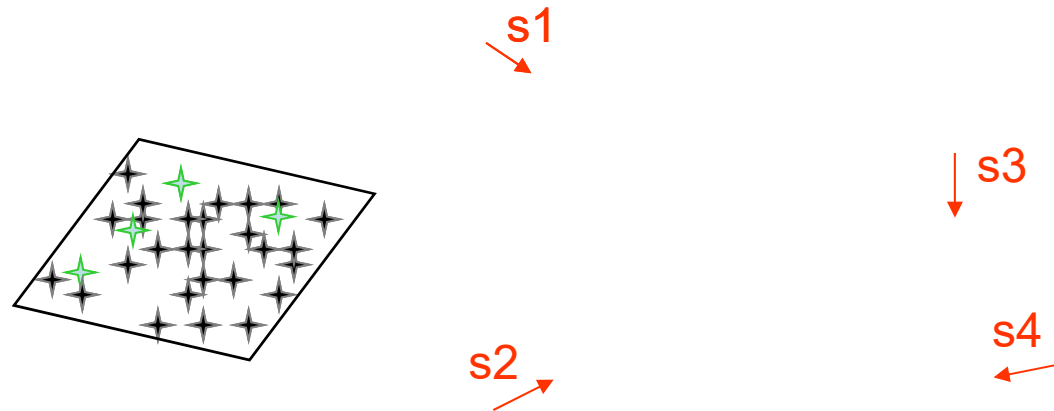


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- GGH Sign 1995
- NTRU Encrypt 1996
- NTRU Sign 2003

# Learning the secret key

Nguyen and Regev 2006



NTRU-251 broken using  $\approx 400$  signatures

GGH-400 broken using  $\approx 160.000$  signatures



# State-of-the-art lattice- based signatures

# Lattice-based signature schemes

Signature scheme	Year	Computational Assumption	ROM?	Tight?	QROM?	Tight?
GPV	2008	SIS	✓	✓	✓	✓
BG	2014	SIS, LWE	✓	x	-	-
TESLA	2017	LWE	✓	✓	✓	✓
GPV-poly	2013	R-SIS	✓	✓	✓	✓
GLP	2012	DCK	✓	x	-	-
BLISS	2013	R-SIS, NTRU	✓	x	-	-



# SIS and LWE

# Small Integer Solutions Problem (SIS)

$$\begin{bmatrix} 3 & 1 & 2 & 4 & 2 & 4 \\ 0 & 3 & 6 & 2 & 0 & 3 \\ 2 & 0 & 4 & 1 & 3 & 2 \end{bmatrix} \cdot \begin{bmatrix} ? \\ ? \\ ? \\ ? \\ ? \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \pmod{7}$$

# Small Integer Solutions Problem (SIS)

$$\begin{bmatrix} 3 & 1 & 2 & 4 & 2 & 4 \\ 0 & 3 & 6 & 2 & 0 & 3 \\ 2 & 0 & 4 & 1 & 3 & 2 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ -1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \pmod{7}$$

# SIS - general

Given:

$$A \leftarrow_{\$} \mathbb{Z}_q^{n \times m} \cdot \text{?} = 0 \pmod{q}$$

Find:

Bound  $\beta$

$$s \in \mathbb{Z}_q^m \text{ with } \|s\| < \beta \text{ and}$$

$$A \cdot s = 0 \pmod{q}$$

# Learning with errors problem (LWE)

3	1	2							
0	3	6							
2	0	4							
4	2	4							
2	0	3							
1	3	2							

·

?
?
?

+

?
?
?
?
?

=

6
1
6
2
5
3

mod 7

$$A \cdot s + e = b \pmod{q}$$

# Learning with errors problem (LWE)

3	1	2				
0	3	6				
2	0	4				
4	2	4				
2	0	3				
1	3	2				

 · 

1
0
1

 + 

1
2
0
1
0
0

 = 

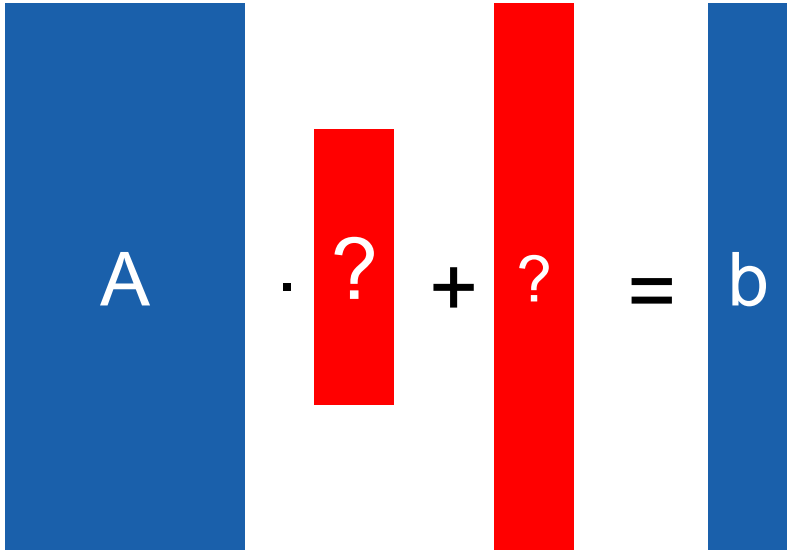
6
1
6
2
5
3

 mod 7

$$A \cdot s + e = b \pmod{q}$$

# LWE - general

Given:


$$A \cdot ? + ? = b \pmod{q}$$
$$A \leftarrow_{\$} \mathbb{Z}_q^{m \times n}$$
$$b \leftarrow \mathbb{Z}_q^m$$

Find:

$$A \cdot s + e = b \pmod{q} \quad (s, e) \leftarrow D_{\sigma}^n \times D_{\sigma}^m$$

# Practical complexity of LWE



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## INFORMATION

Unfortunately, the creation of the LWE instances was bugged and resulted in unbreakable

[https://latticechallenge.org/lwe\\_challenge](https://latticechallenge.org/lwe_challenge)

(grey) instances will be added soon. Sorry for any inconveniences.

## INTRODUCTION

Welcome to the Learning With Errors (LWE) challenge.

The LWE problem is to recover  $\mathbf{s}$ , given an instance  $(\mathbf{A}, \mathbf{b})$ , where  $\mathbf{A}$  is an  $m \times n$  matrix over  $\mathbb{Z}_q$  and  $\mathbf{b} = \mathbf{A}\mathbf{s} + \mathbf{e}$  is a vector of length  $m$  over  $\mathbb{Z}_q$ . Both the matrix  $\mathbf{A}$  and the target vector  $\mathbf{s}$

## SUBMISSION

Submission

Format of Challenge Files

Toy Challenges in  
Dimension:

$n=2, \alpha = 0.005$

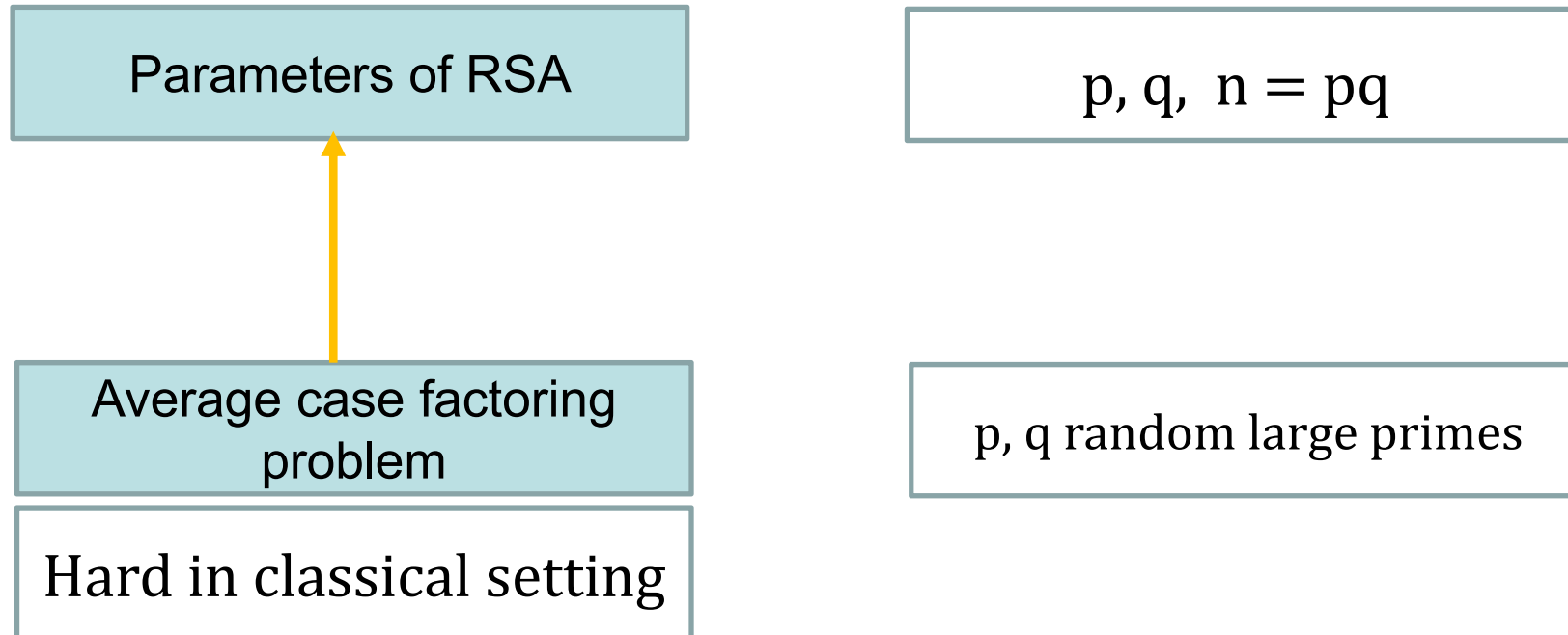
$n=5, \alpha = 0.010$



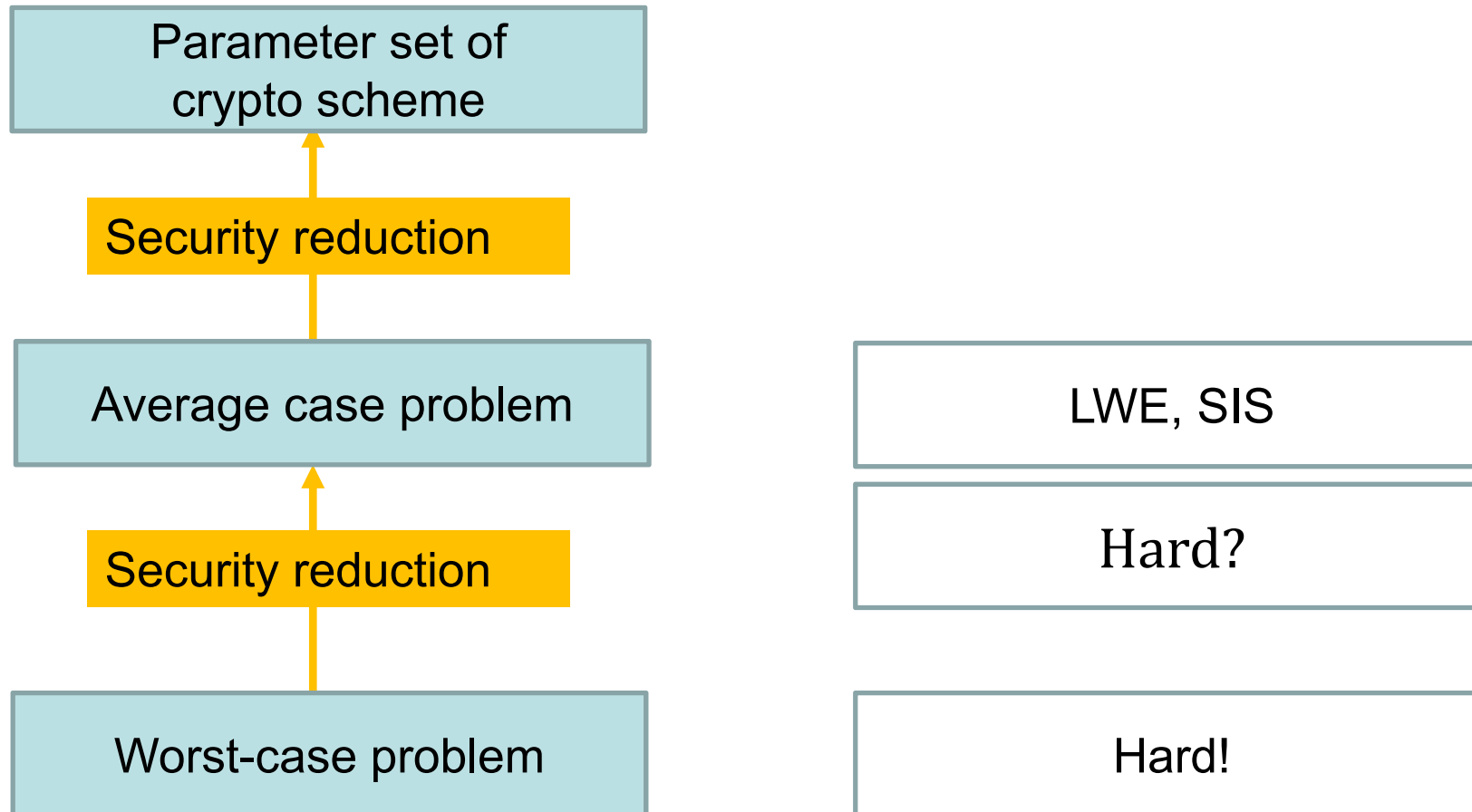


# Selecting secure parameters

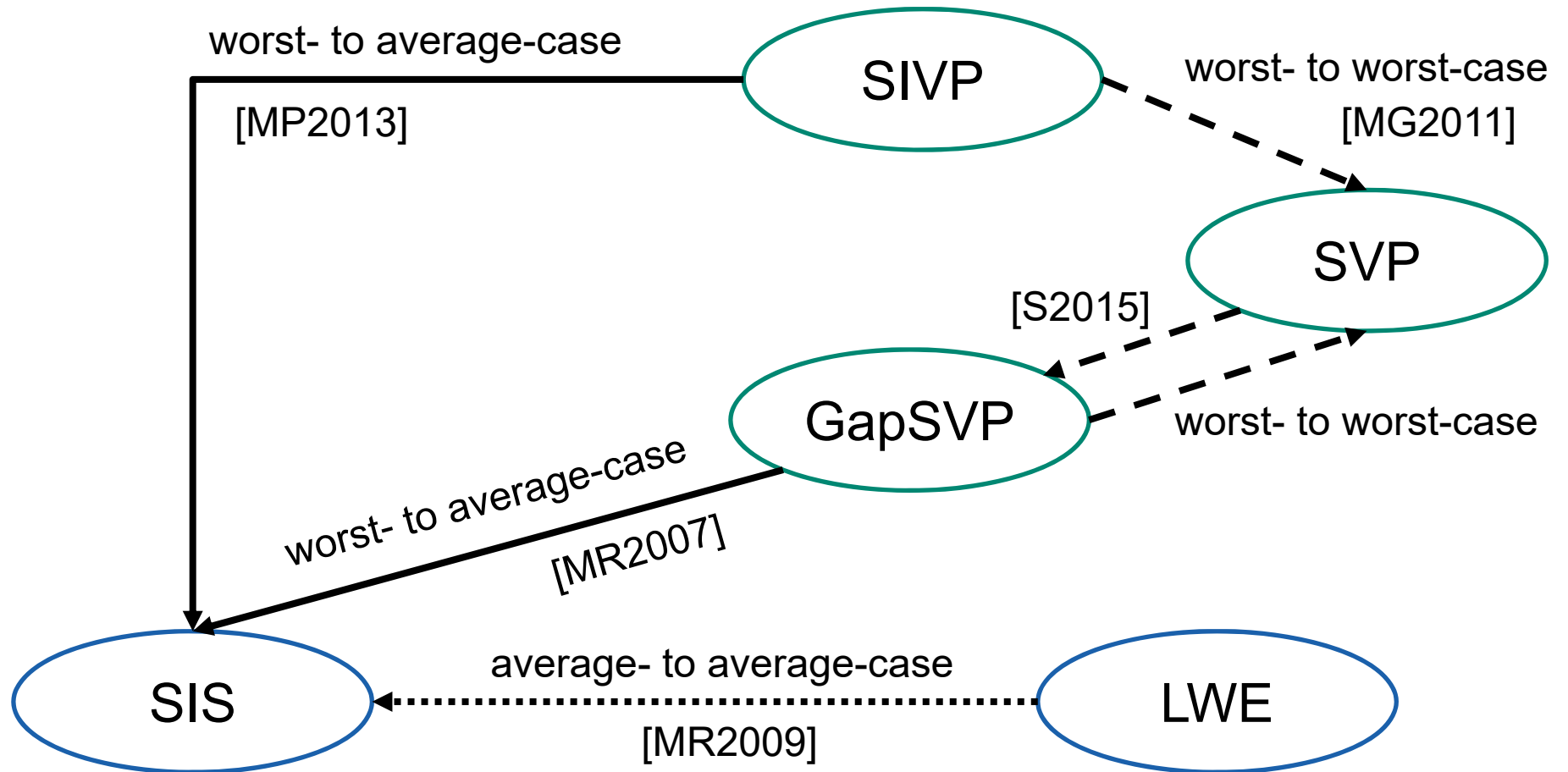
# Selecting secure RSA parameters



# Secure parameters for lattice-based schemes



# Reductions between lattice problems





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# TESLA

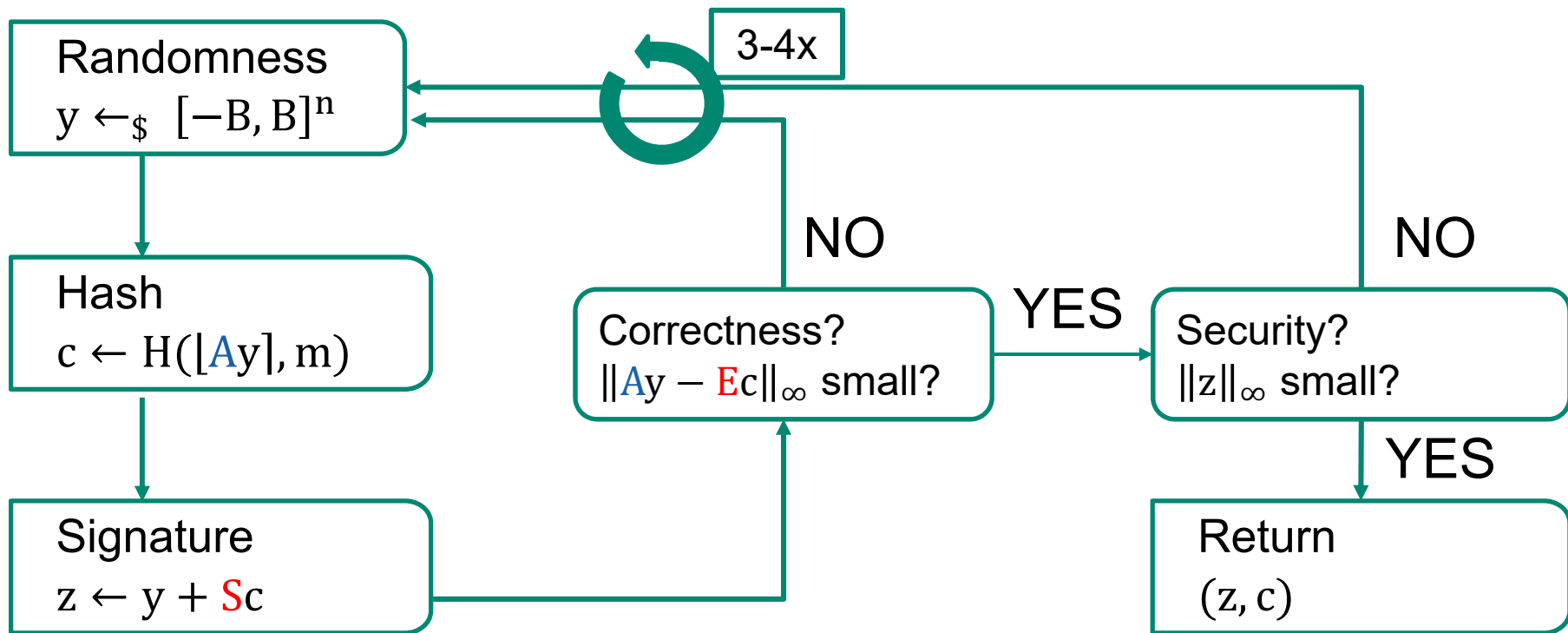
# TESLA signature scheme

Sign(sk, m):

$$A \leftarrow_{\$} \mathbb{Z}_q^{m \times n}$$

$$sk = (S, E) \leftarrow_{\sigma} \mathbb{Z}_q^{n \times n} \times \mathbb{Z}_q^{m \times n}$$

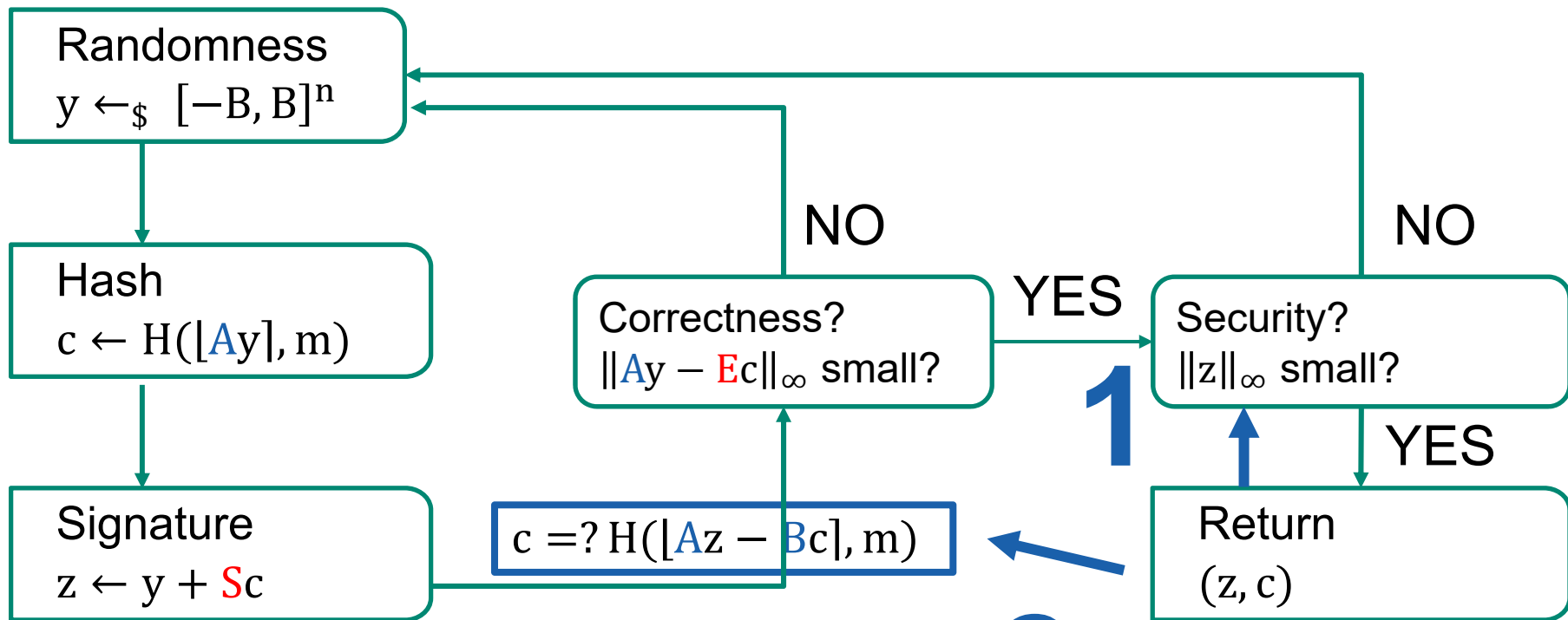
$$pk = (A, B = AS + E \text{ mod } q)$$



# TESLA - verification

$$\begin{aligned}
 A &\leftarrow_{\$} \mathbb{Z}_q^{m \times n} \\
 \text{sk} &= (S, E) \leftarrow_{\sigma} \mathbb{Z}_q^{n \times n} \times \mathbb{Z}_q^{m \times n} \\
 \text{pk} &= (A, B = AS + E \text{ mod } q)
 \end{aligned}$$

Sign(sk, m):



# ring-TESLA



$$\begin{aligned} A &\leftarrow_{\$} \mathbb{Z}_q^{m \times n} \\ \text{sk} &= (\mathbf{S}, \mathbf{E}) \leftarrow \mathbb{Z}_q^{n \times n} \times \mathbb{Z}_q^{m \times n} \\ \text{pk} &= (A, B = \mathbf{AS} + \mathbf{E}) \end{aligned}$$

Most expensive  
operation

$$\begin{aligned} a_1, a_2 &\leftarrow \mathbb{Z}_q[x] / \langle x^n + 1 \rangle \\ \text{sk} &= (\mathbf{s}, \mathbf{e}_1, \mathbf{e}_2) \leftarrow \mathbb{Z}_q[x] / \langle x^n + 1 \rangle \\ \text{pk} &= (a_1, a_2, b_1 = a_1 \mathbf{s} + \mathbf{e}_1, b_2 = a_2 \mathbf{s} + \mathbf{e}_2) \end{aligned}$$



# TESLA signature scheme – Performance



Scheme	Cyclecounts [k-cycles]		Sizes [kB]			Security
	Sign	Verify	pk	sk	sig.	[bit]
<b>GPV</b>	312,800	50,600	27,840	12,064	29	96
<b>BG</b>	1,204	335	1,582	891	1.5	97
<b>TESLA</b>	41,604	5,017	16,406	9,986	1.9	96
<b>GPV-poly</b>	80,500	11,500	55	26	32	96
<b>GLP</b>	452	34	1.5	0.25	1.12	80
<b>BLISS</b>	358	102	0.87	0.25	0.63	128
<b>RSA-2048</b>	5,347	76	0.25	0.25	0.25	112
<b>ECDSA P256</b>	388	920	0.06	0.09	0.06	128



# State-of-the-art lattice- based PK encryption

# Lattice-based PK encryption schemes

Encryption scheme	Year	Computational Assumption	CPA/CCA?	Security reduction
<b>LWE Regev</b>	2005	LWE	CPA	yes
<b>LARA</b>	2016	(R-)LWE	CPA, CCA1, CCA2	Yes
<b>NTRU</b>	1998	NTRU	CPA (CCA1)	-
<b>LP</b>	2011	(R-)LWE	CPA	Yes
<b>CCA-MP</b>	2012	LWE	CCA1	Yes
<b>CCA-Peikert</b>	2009	LWE	CCA1/2	Yes

# LARA – Performance (128-bit Security)

El Bansarkhani et al.

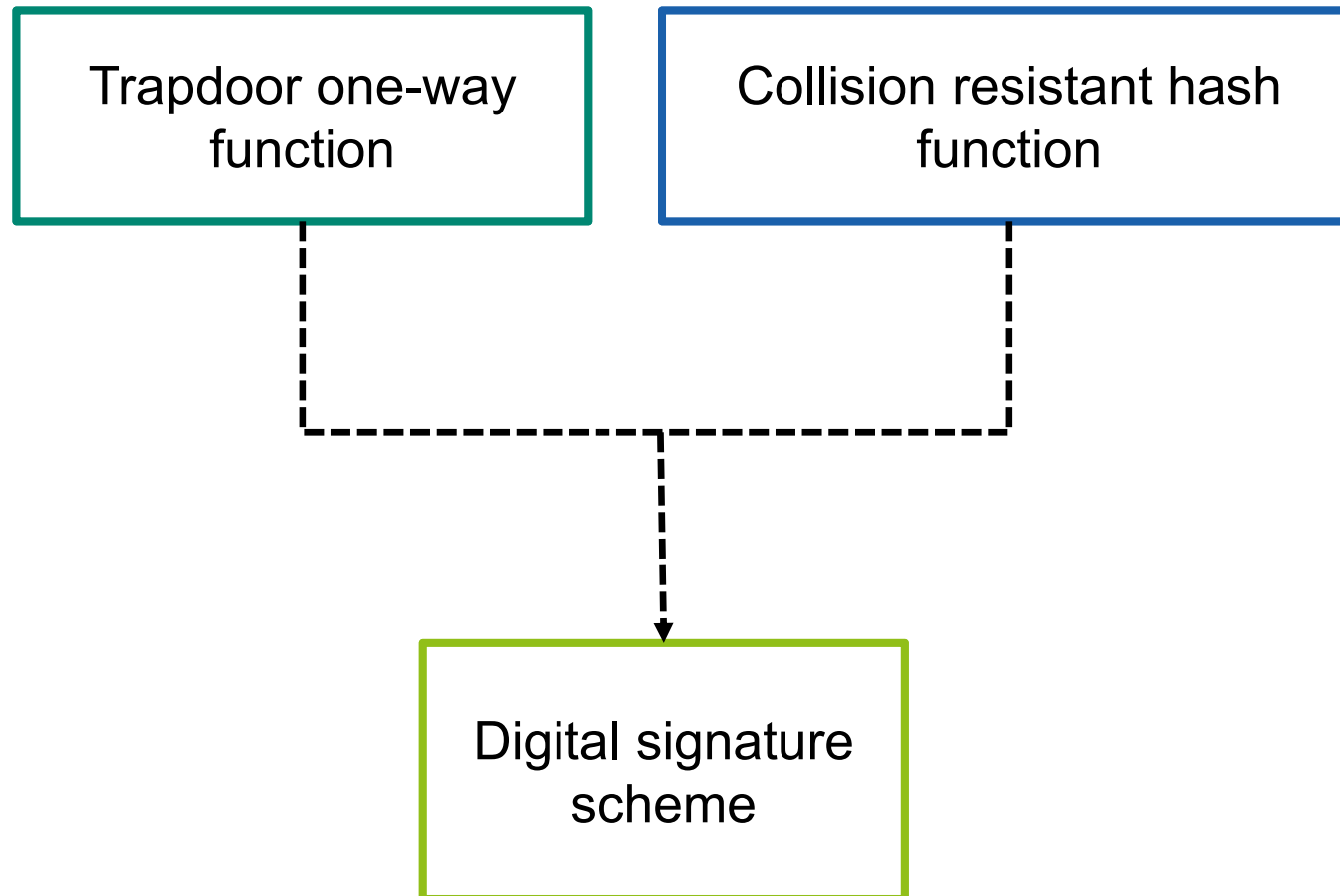


Scheme	Cyclecounts/message bit		Sizes [kB]		Ciphertext expansion
	Encrypt	Decrypt	pk	sk	
LARA-CPA	22	15	1.25	0.28	2.8
LARA-CCA1	29	22	1.19	0.28	3.1
LARA-CCA2	50	42	1.19	0.28	3.1
Linder-Peikert-CPA	444	65	1.25	0.28	40
NTRU-CCA2	138	158	0.69	0.64	8
RSA-4096	51	2992	0.25	0.25	1



# Hash-based signatures

# Typical construction





# Trapdoor one-way functions hard to construct but not required



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# Hash-based Signatures

## Merkle (1979/1989)

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### A CERTIFIED DIGITAL SIGNATURE

*Ralph C. Merkle  
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3333 Coyote Hill Road,  
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(Subtitle: That Antique Paper from 1979)*



#### Abstract

A practical digital signature system based on a conventional encryption function which is as secure as the conventional encryption function is described. Since certified conventional systems are available it can be implemented quickly, without the several years delay required for certification of an untested system.

**Key Words and Phrases:** Public Key Cryptosystem, Digital Signatures, Cryptography, Electronic Signatures, Receipts, Authentication, Electronic Funds Transfer.

CR categories: 3.56, 3.57, 4.9




# Lamport-Diffie OTSS

Lamport, Diffie (1976)



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Example:

  $x_1(0), x_1(1), x_2(0), x_2(1), x_3(0), x_3(1)$

signing strings

0 1 1 0 0 1

1 1 1 0 1 0

of length 3

0 0 1 1 1 1

$\downarrow H$

0 1 0 0 1 1

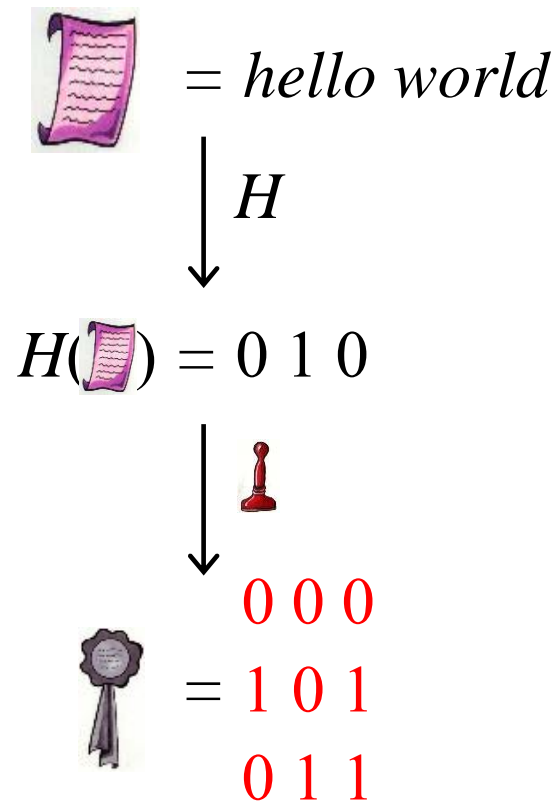
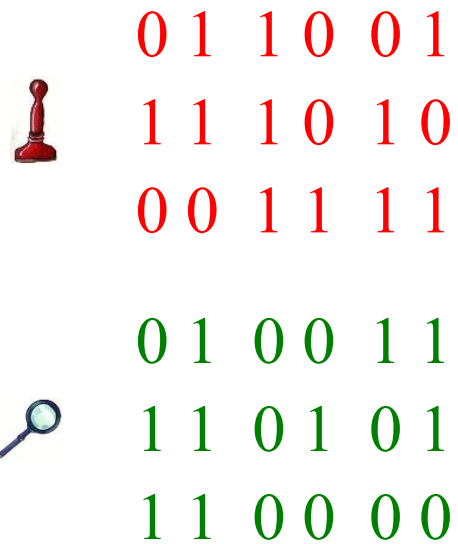
1 1 0 1 0 1

1 1 0 0 0 0

  $y_1(0), y_1(1), y_2(0), y_2(1), y_3(0), y_3(1)$

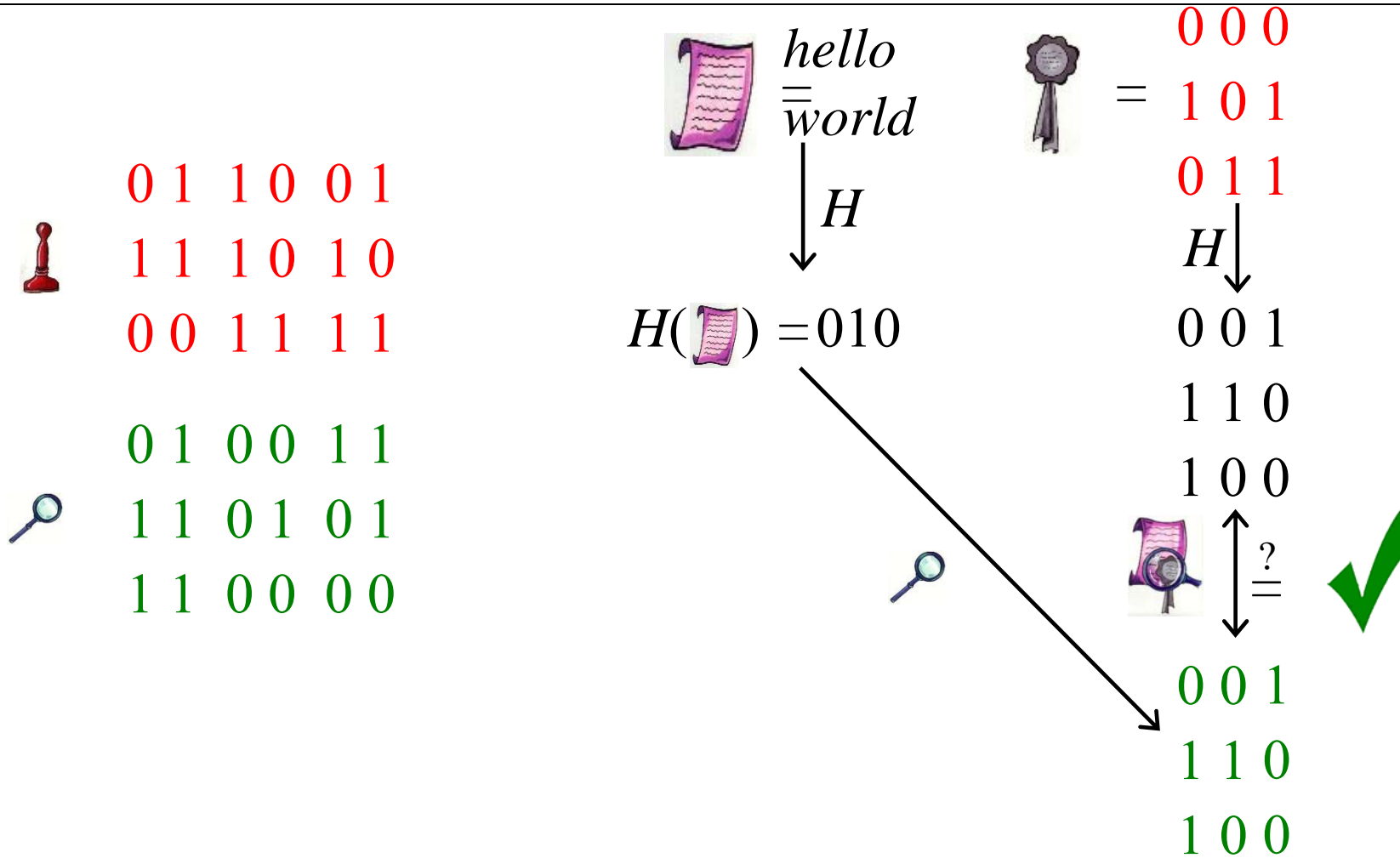
# Lamport-Diffie OTSS

Lamport, Diffie (1976)



# Lamport-Diffie OTSS

Lamport, Diffie (1976)



# Merkle Signature Scheme



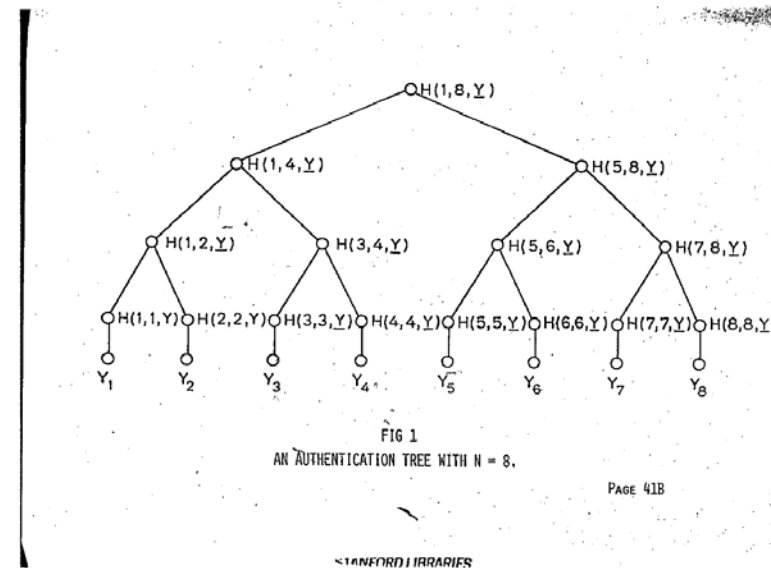
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Lamport-Diffie OTSS:

One key pair ( ,  ) per signature

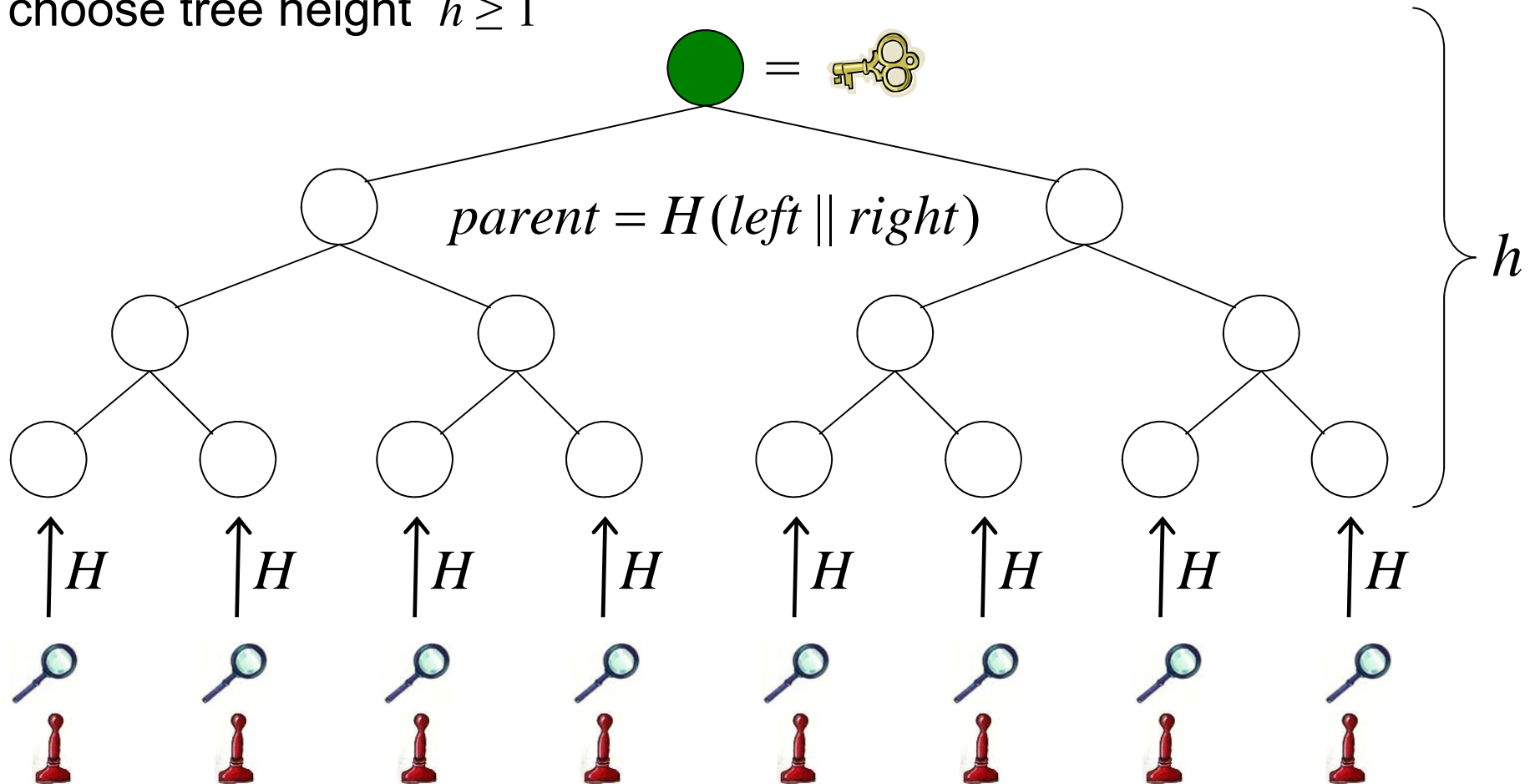
Hash tree:

Reduces validity of many  
verification keys to one public  
key: root of tree

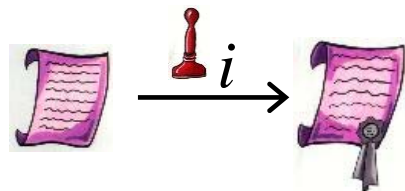
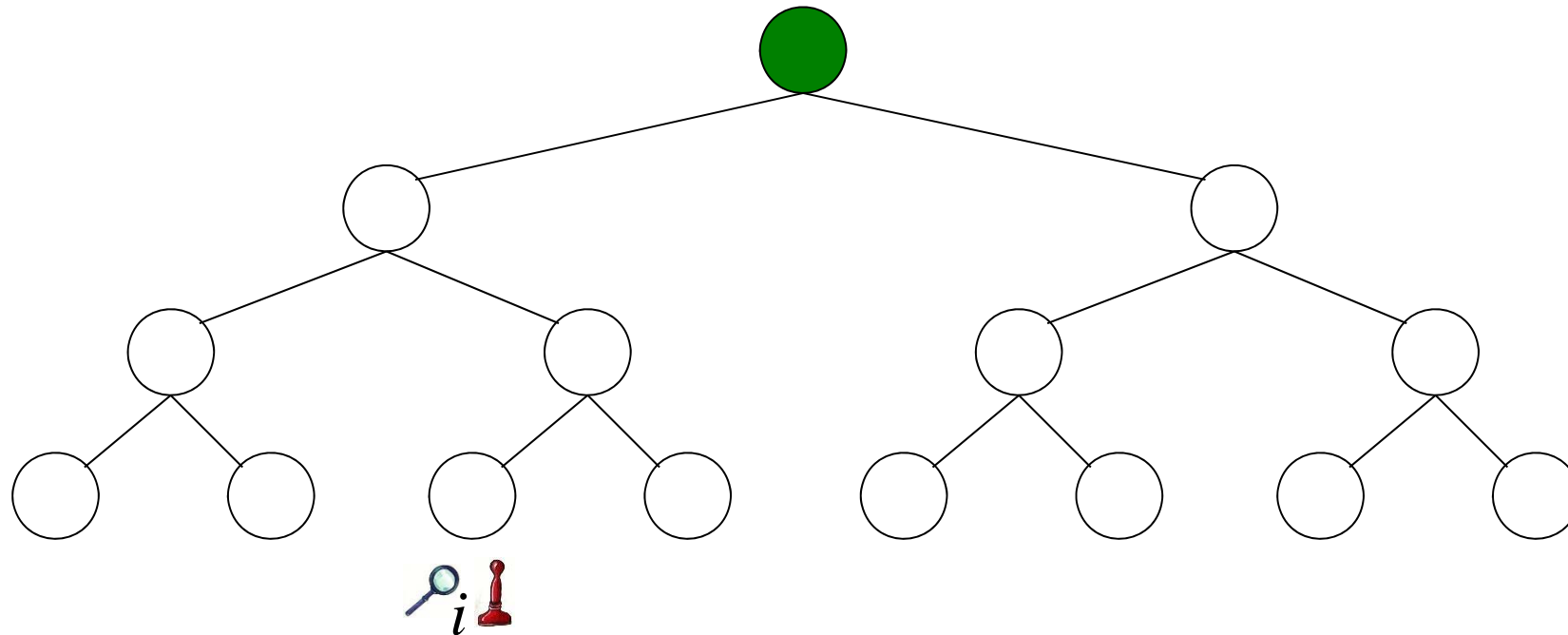


# Merkle Signature Scheme — Key Generation

choose tree height  $h \geq 1$

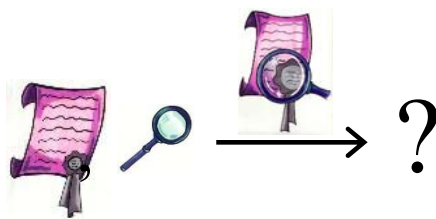
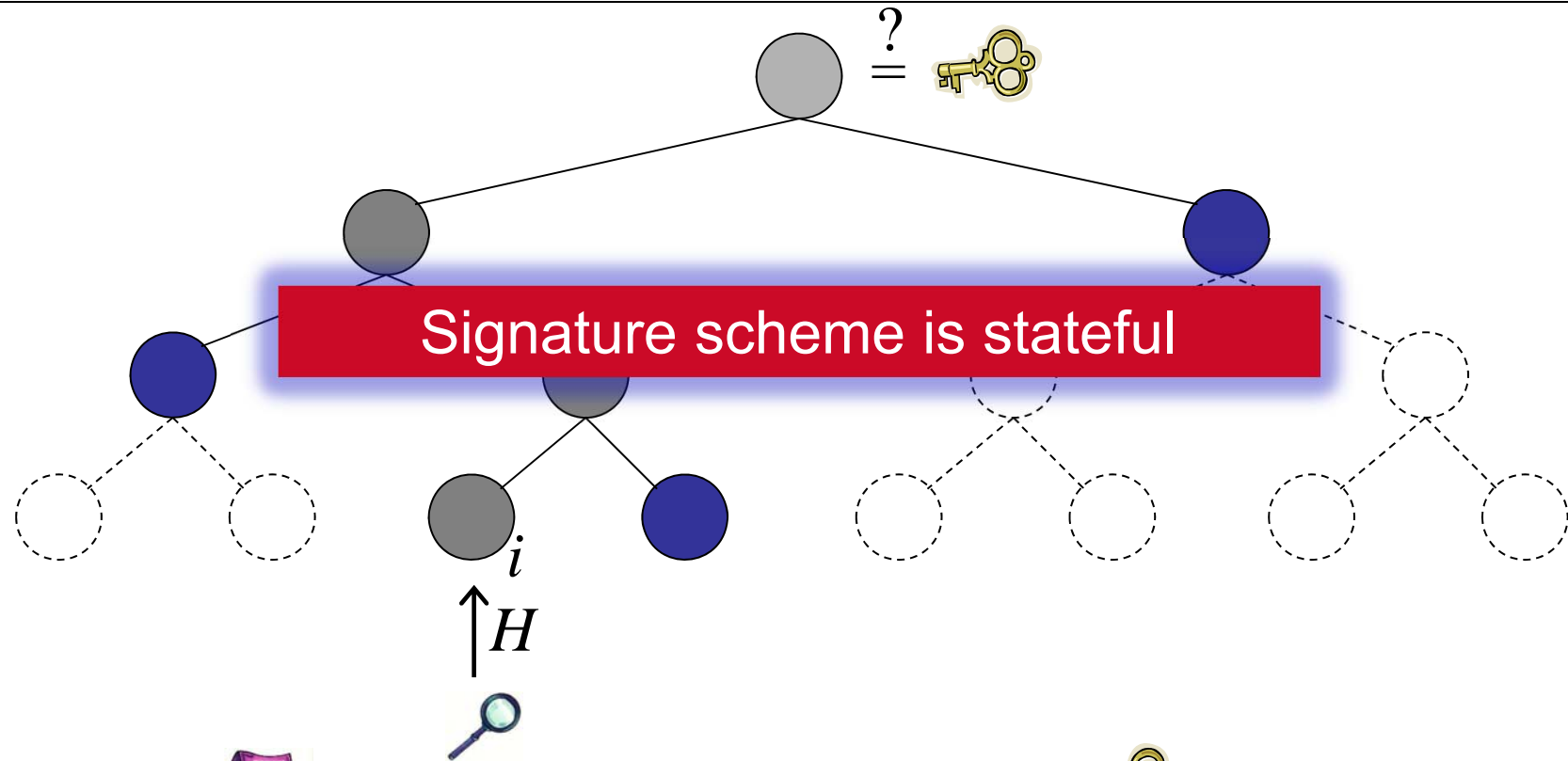


# Merkle Signature Scheme — Signing



Signature =  $(i, \text{document}, \text{key}, \text{leaf}_1, \text{leaf}_2, \text{leaf}_3)$

# Merkle Signature Scheme — Verifying



Public key = 

Signature =  $(i, \text{document icon}, \text{magnifying glass icon}, \text{blue circle}, \text{blue circle}, \text{blue circle})$



# **XMSS:**

## **A practical signature template with minimal security assumptions**

**J.B., Carlos Coronado Garcia, Erik Dahmen,  
Andreas Hülsing**



# XMSS improves

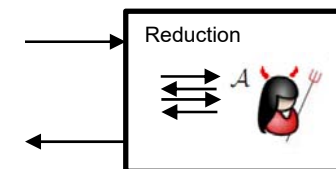
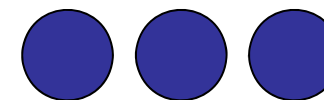
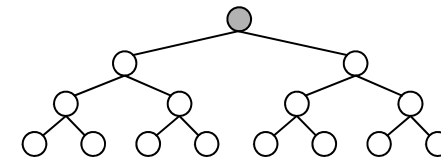
Public key generation time

Private key size

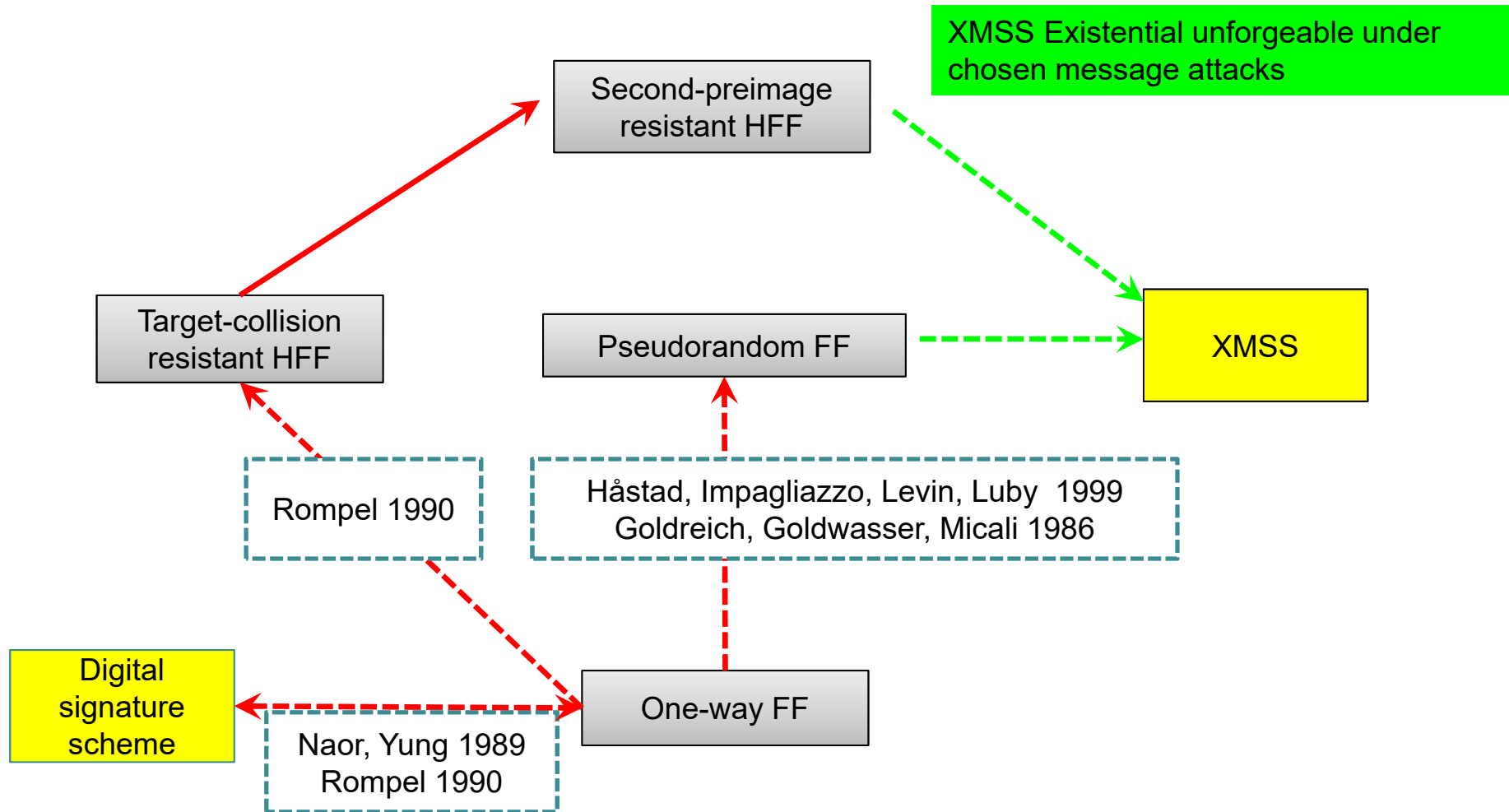
Signature size

Authentication path generation  
time and space

Provable security



# XMSS has minimal security requirements

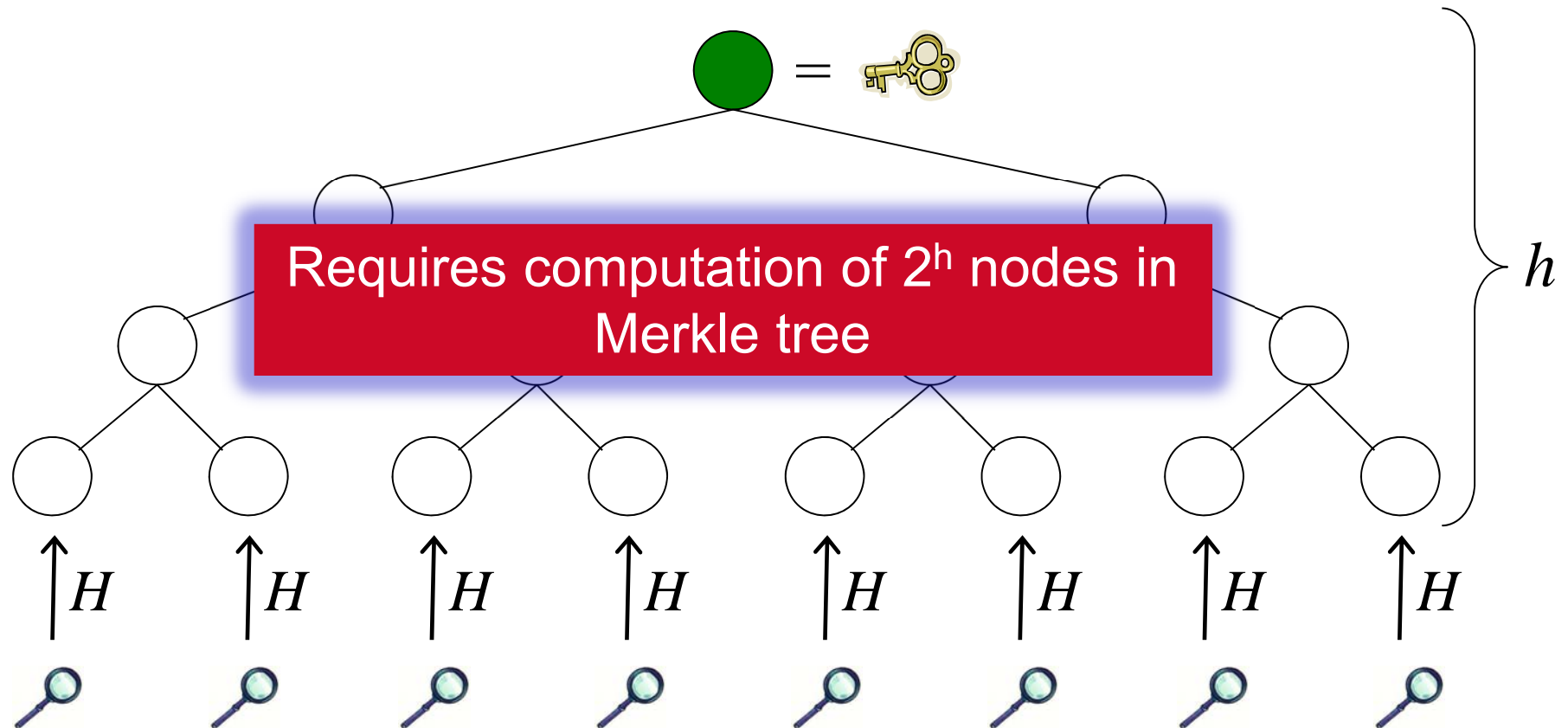




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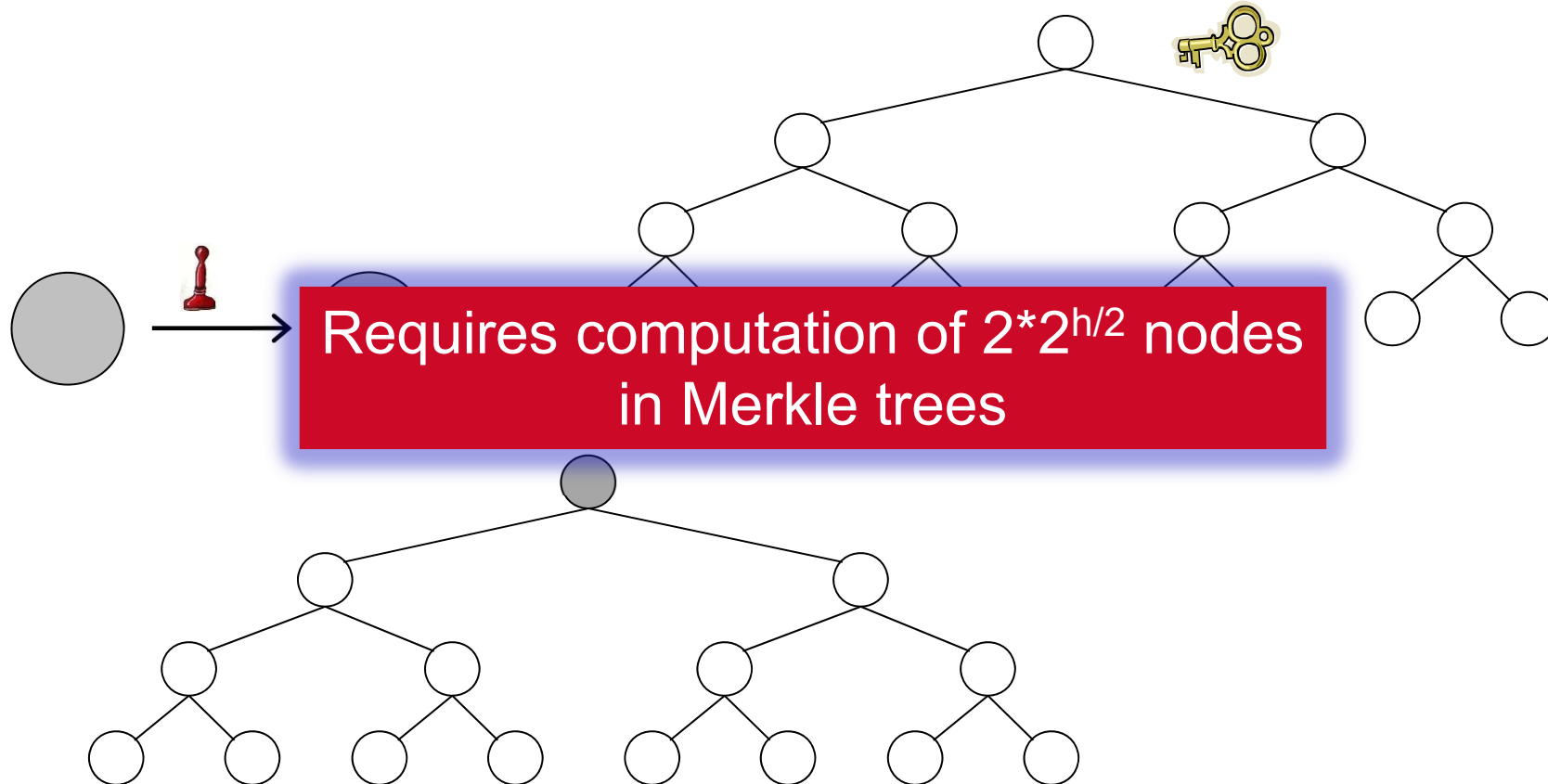
# Improved public key generation: tree chaining

# XMSS Public Key Generation

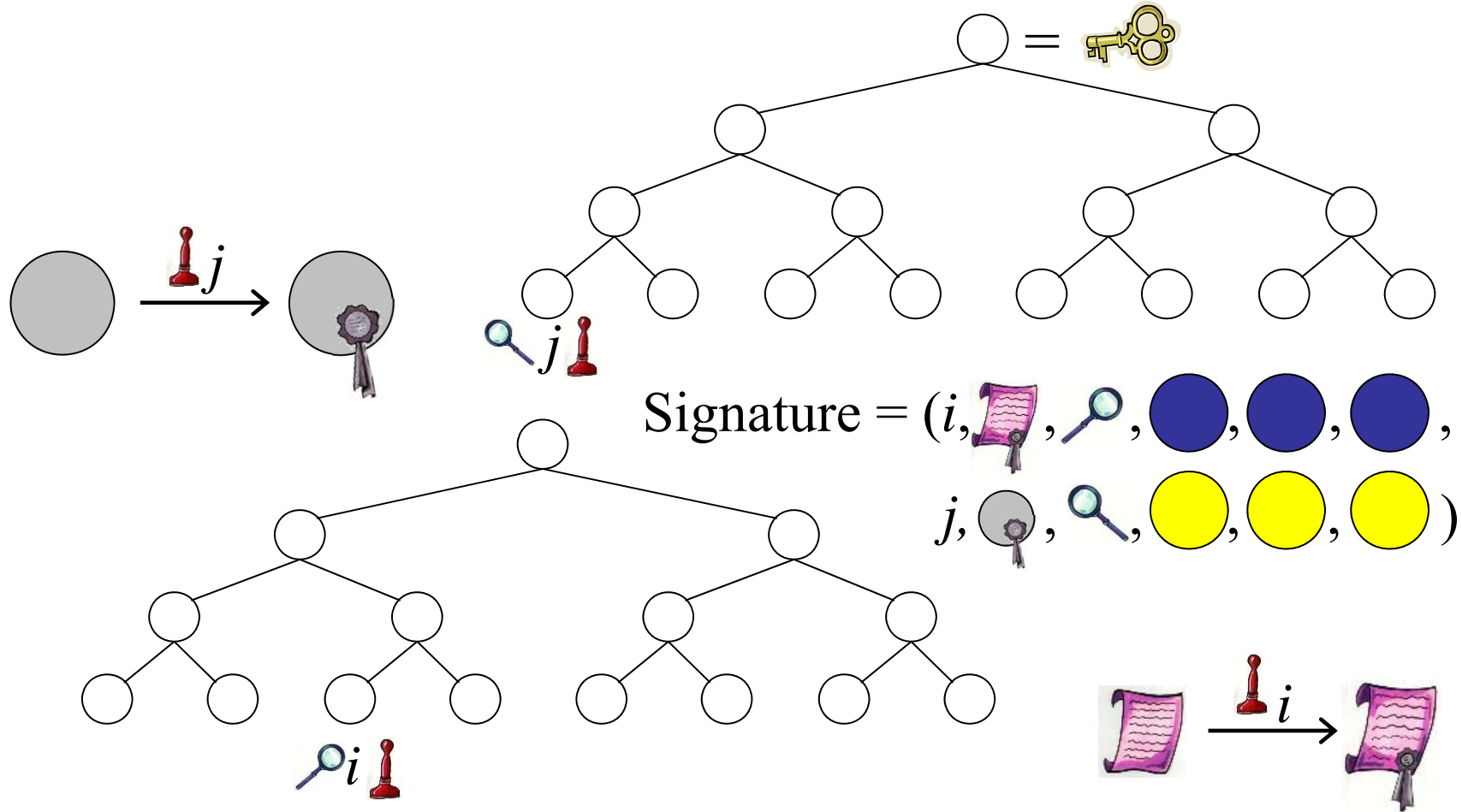


# Two Levels

## Key generation



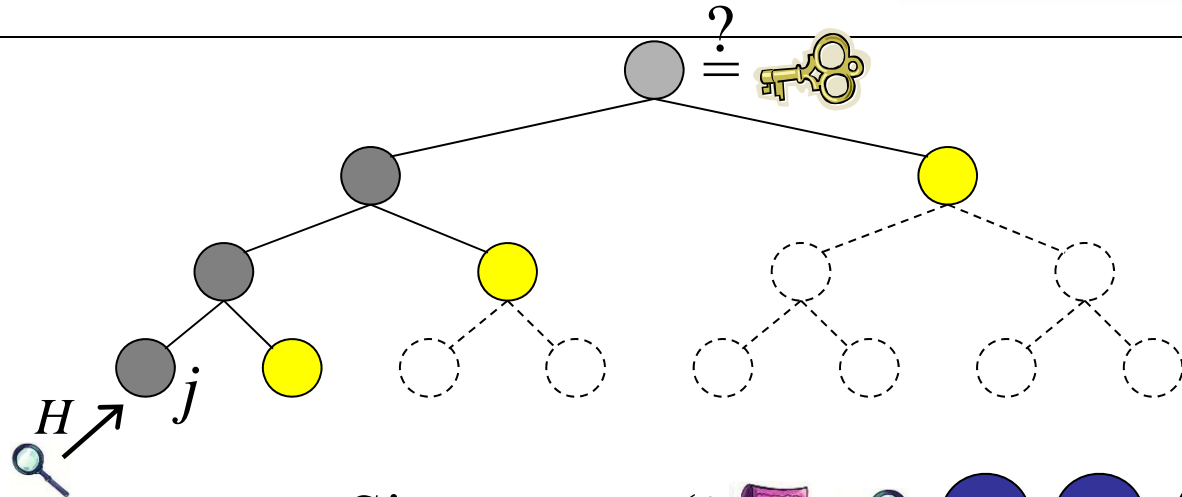
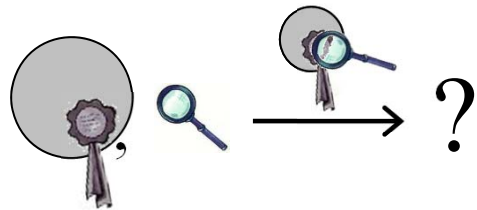
# Two Levels Signing



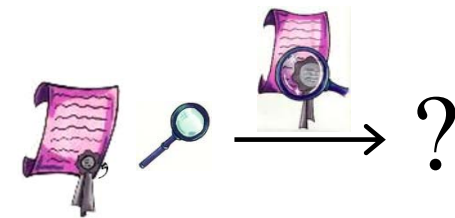
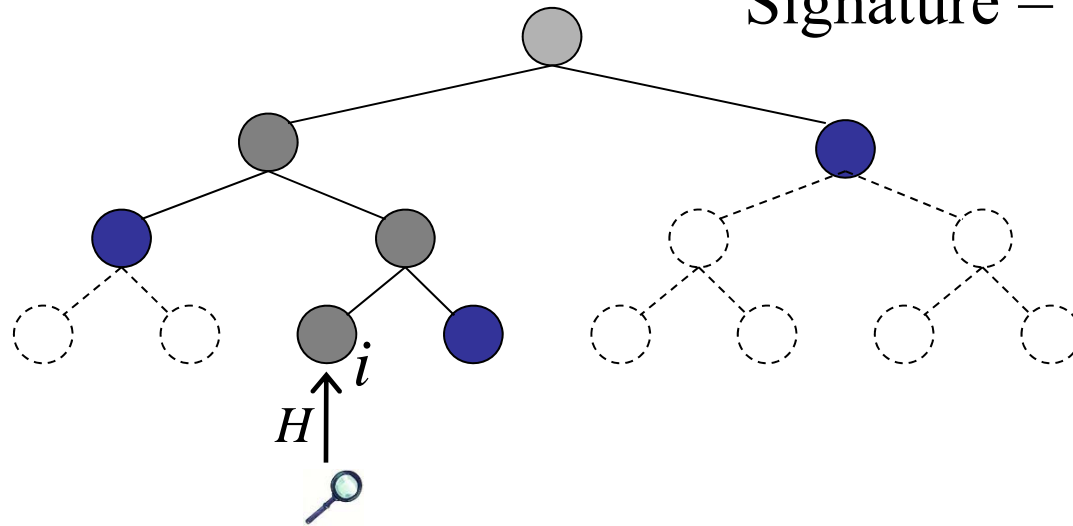
# Two Levels

## Verifying

Public Key = 



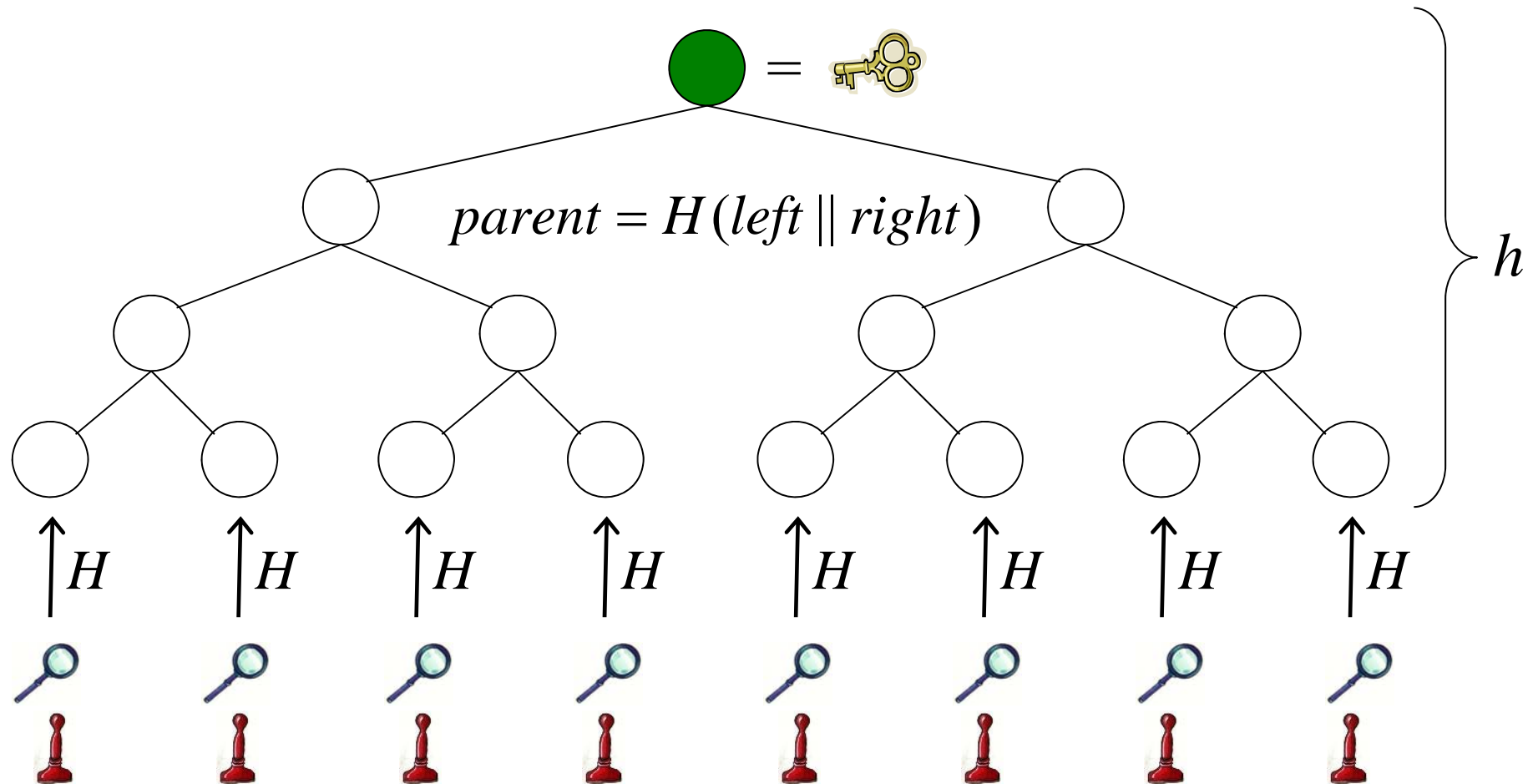
Signature =  $(i, \text{document icon}, \text{magnifying glass}, \text{blue circle}, \text{blue circle}, \text{blue circle}, j, \text{document icon}, \text{magnifying glass}, \text{yellow circle}, \text{yellow circle}, \text{yellow circle})$

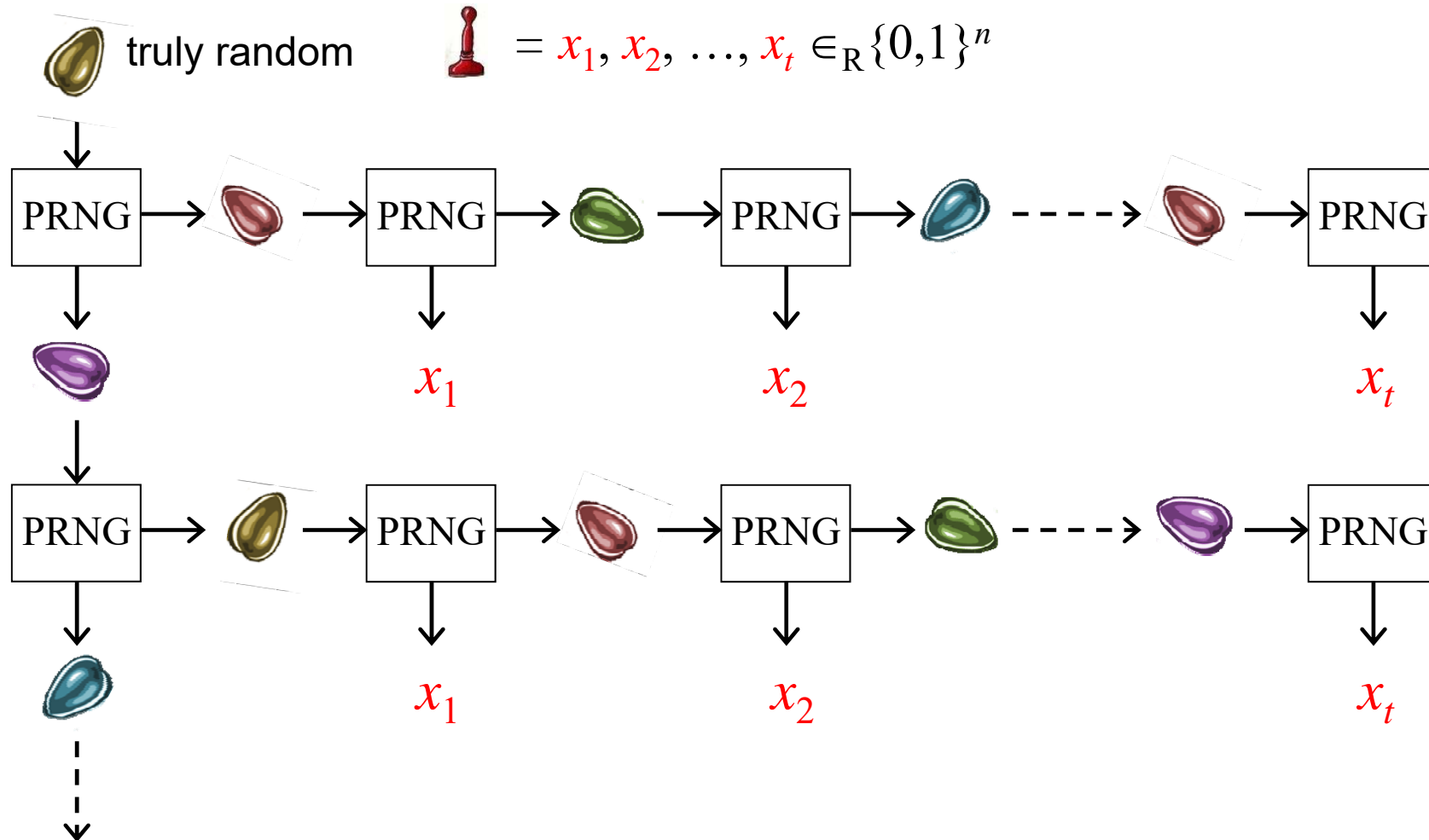




# Improved secret key size: pseudo-random generation









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# Smaller signatures

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# Smaller signatures: Winternitz OTS (WOTS) / WOTS+

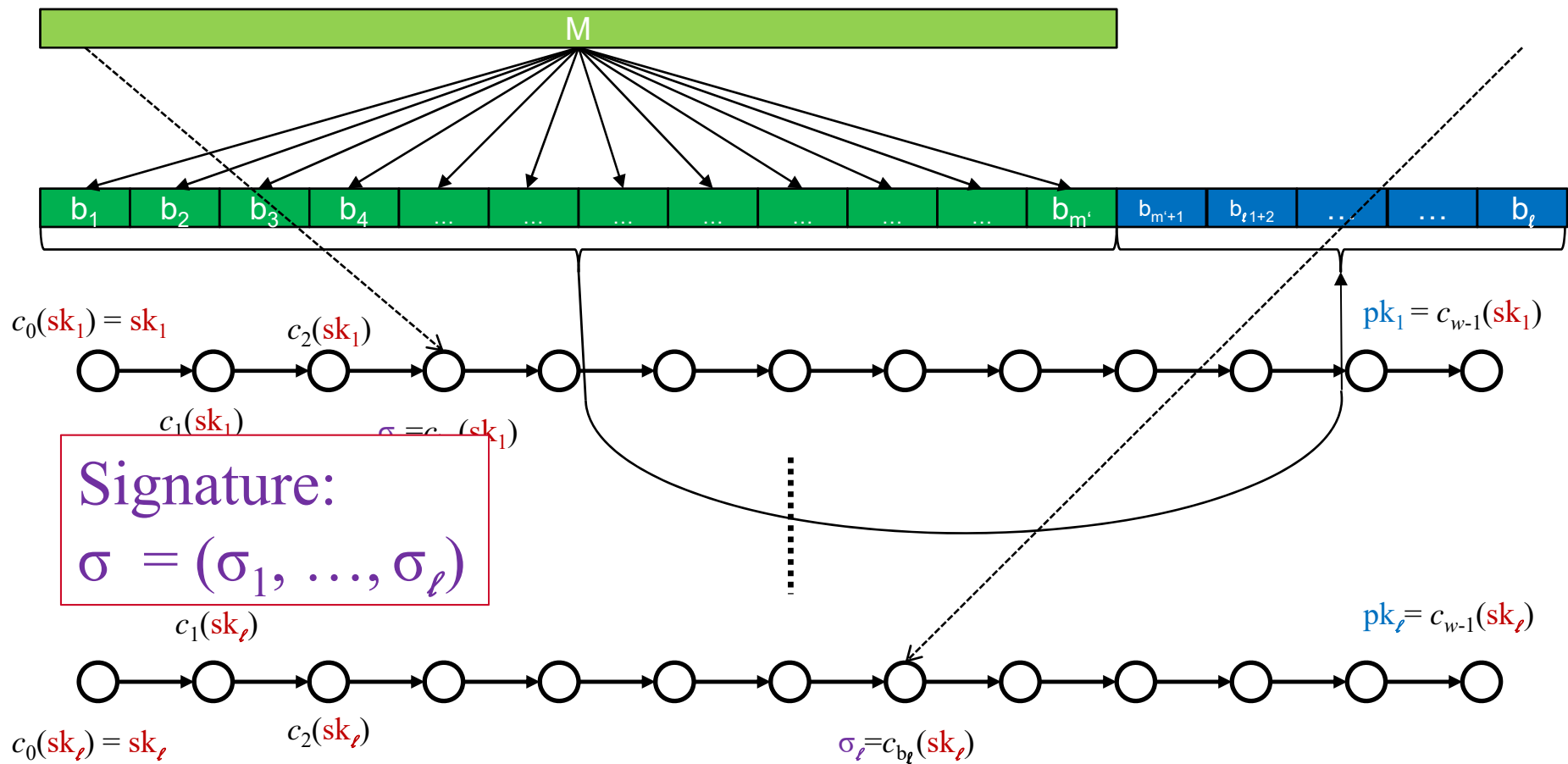
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**Initial idea:** Winternitz (Mer89)

**WOTS+:** Hülsing (Hül13)  
Requires second preimage resistant undetectable one-way function family.

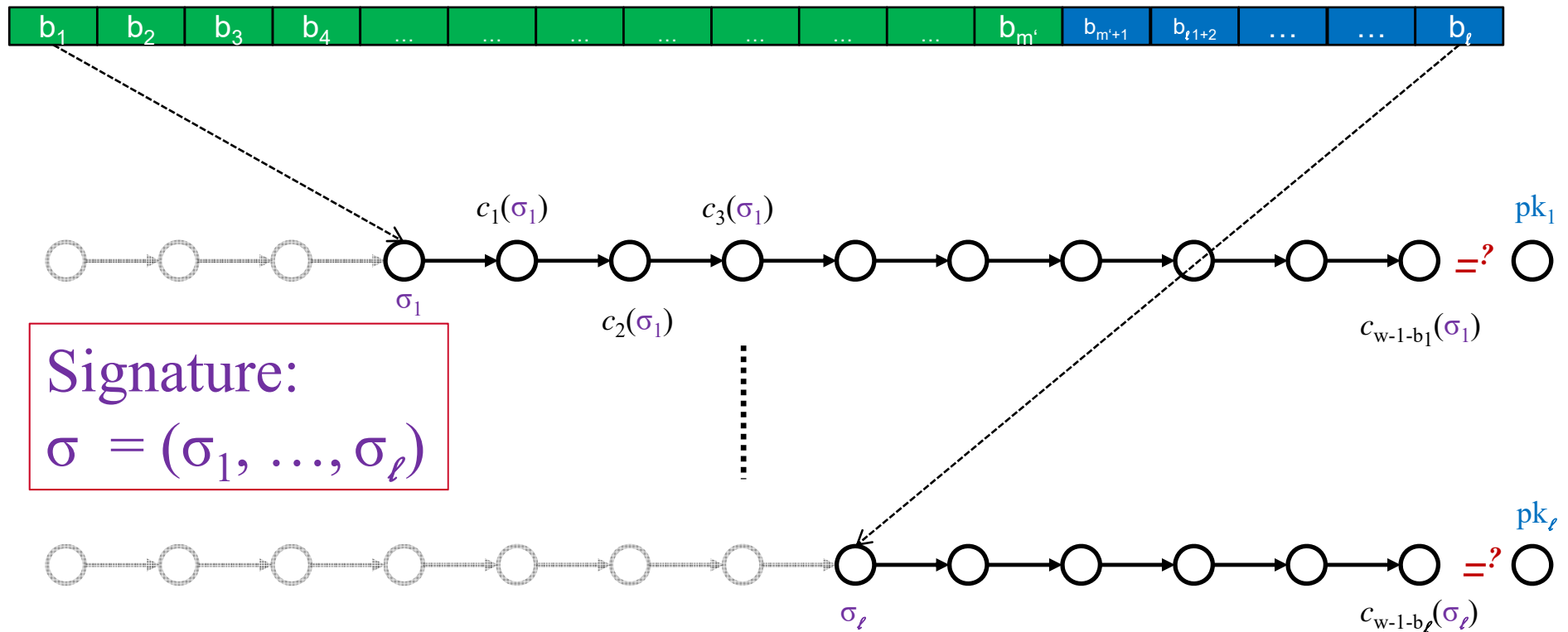
# WOTS+ Signature Generation



# WOTS+ Signature Verification



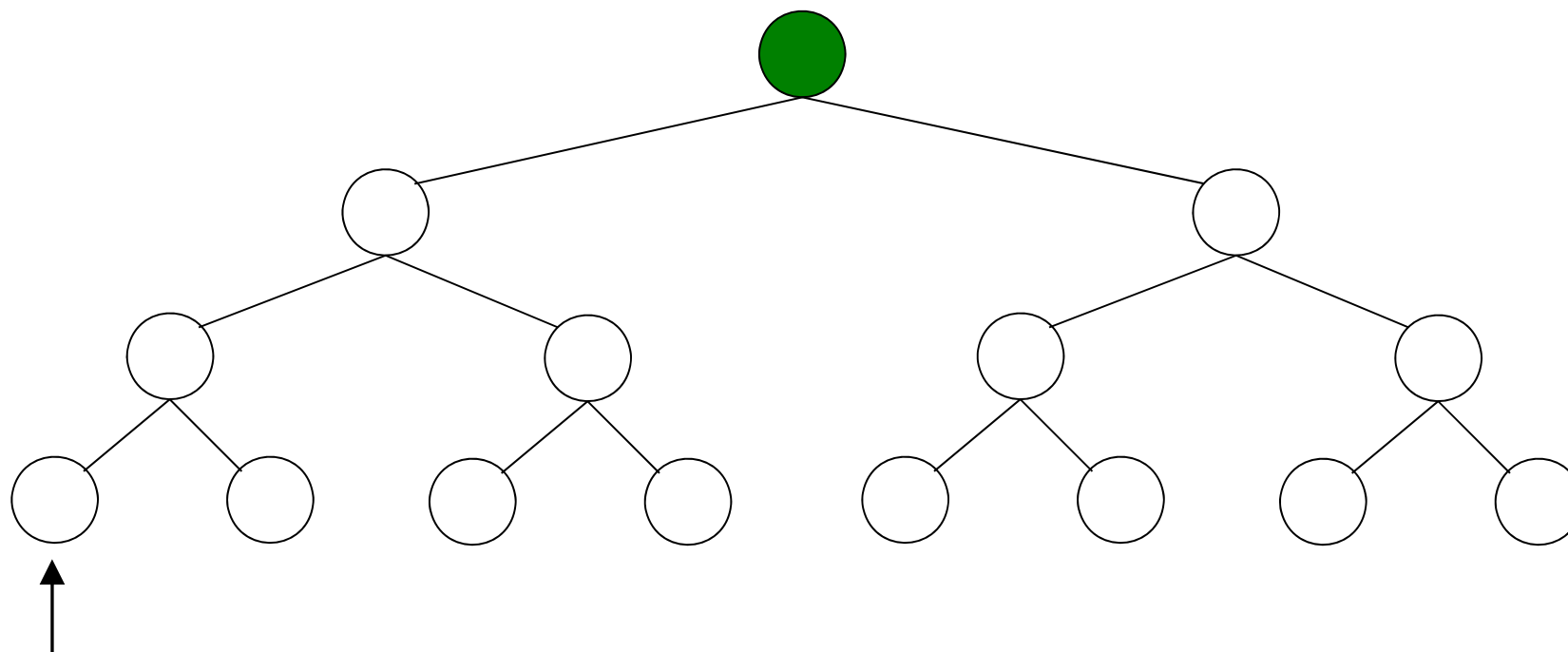
Verifier knows:  $M, n, w, c$





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# Authentication path computation



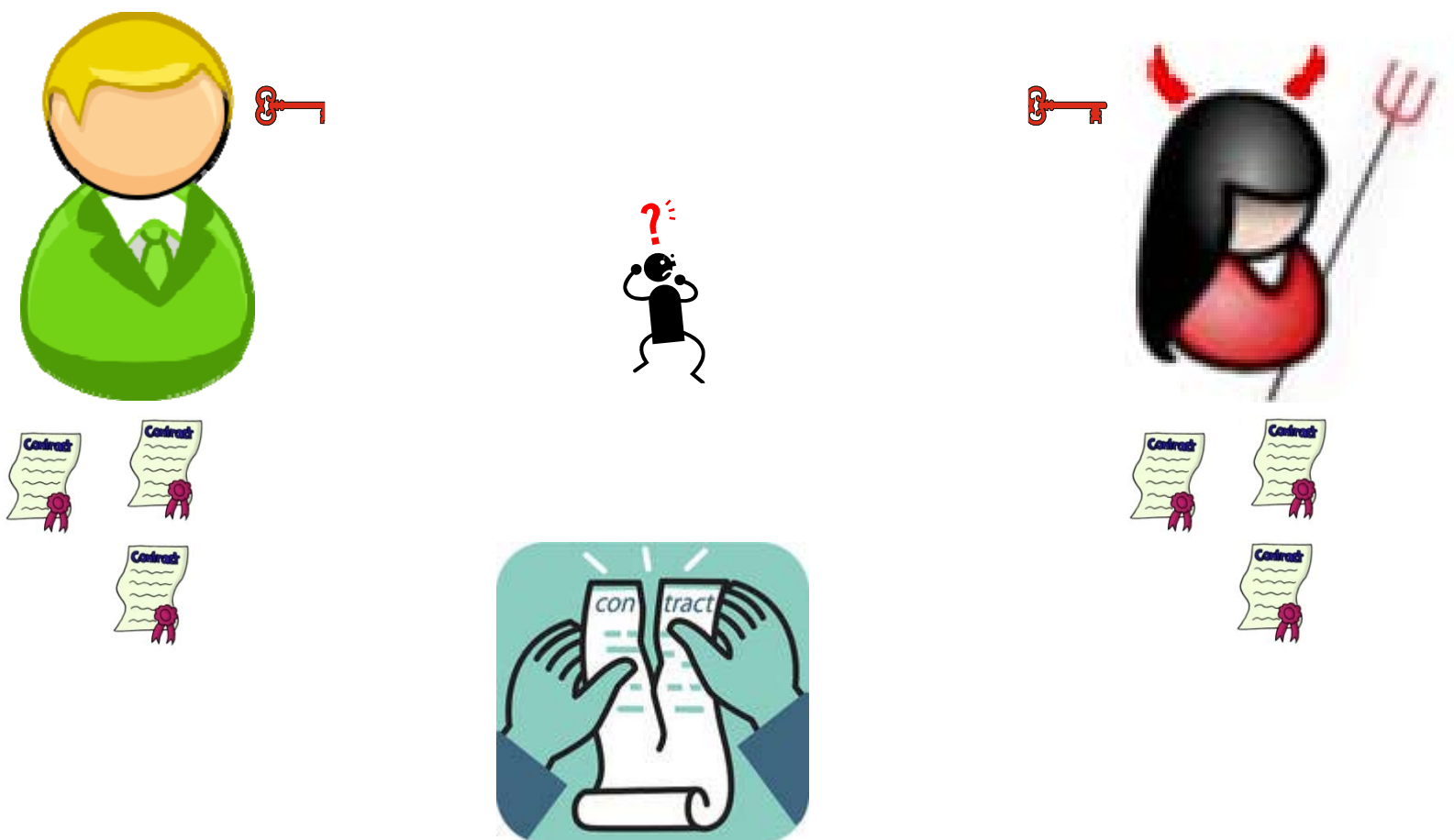




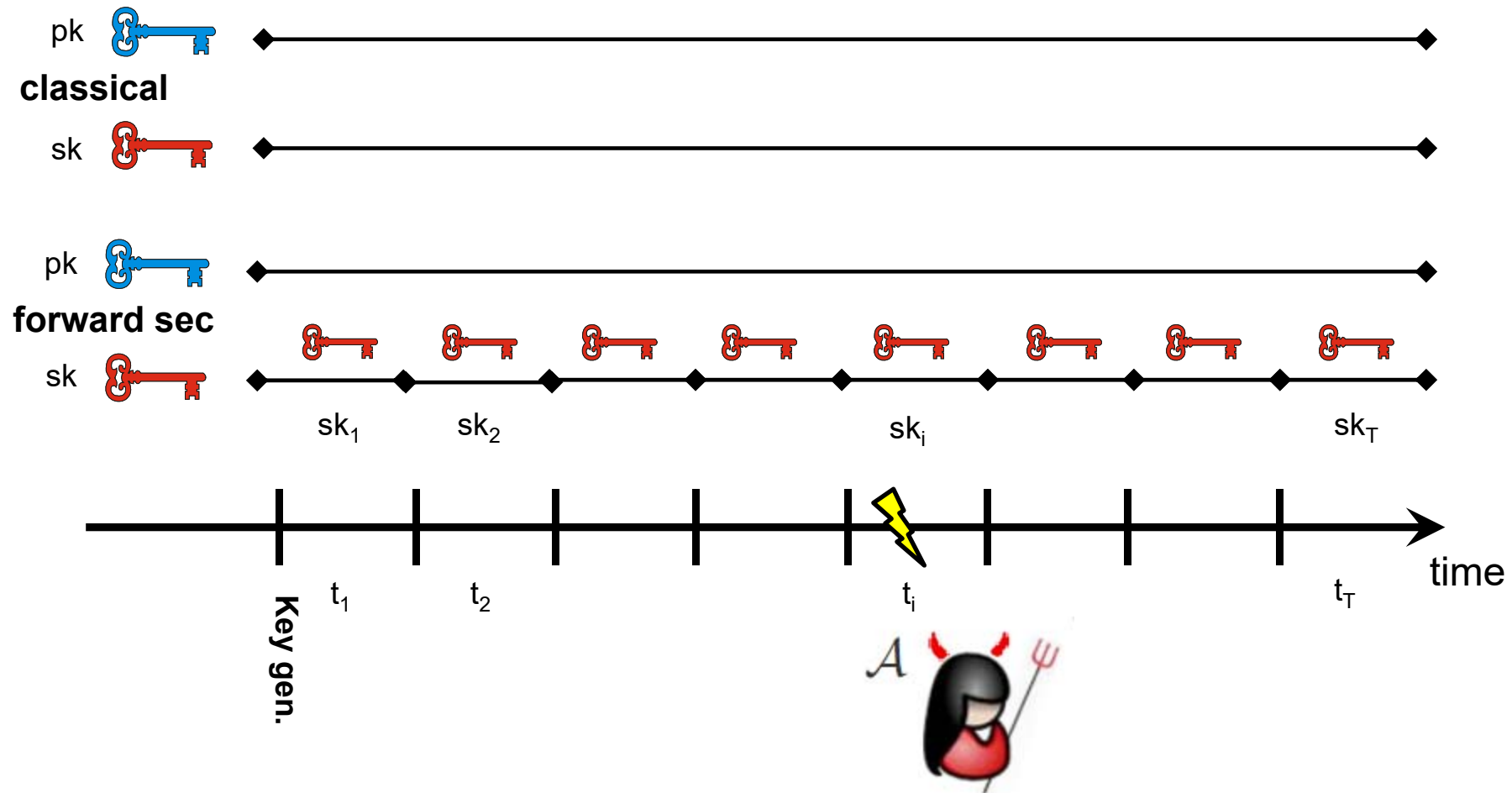
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# Forward security



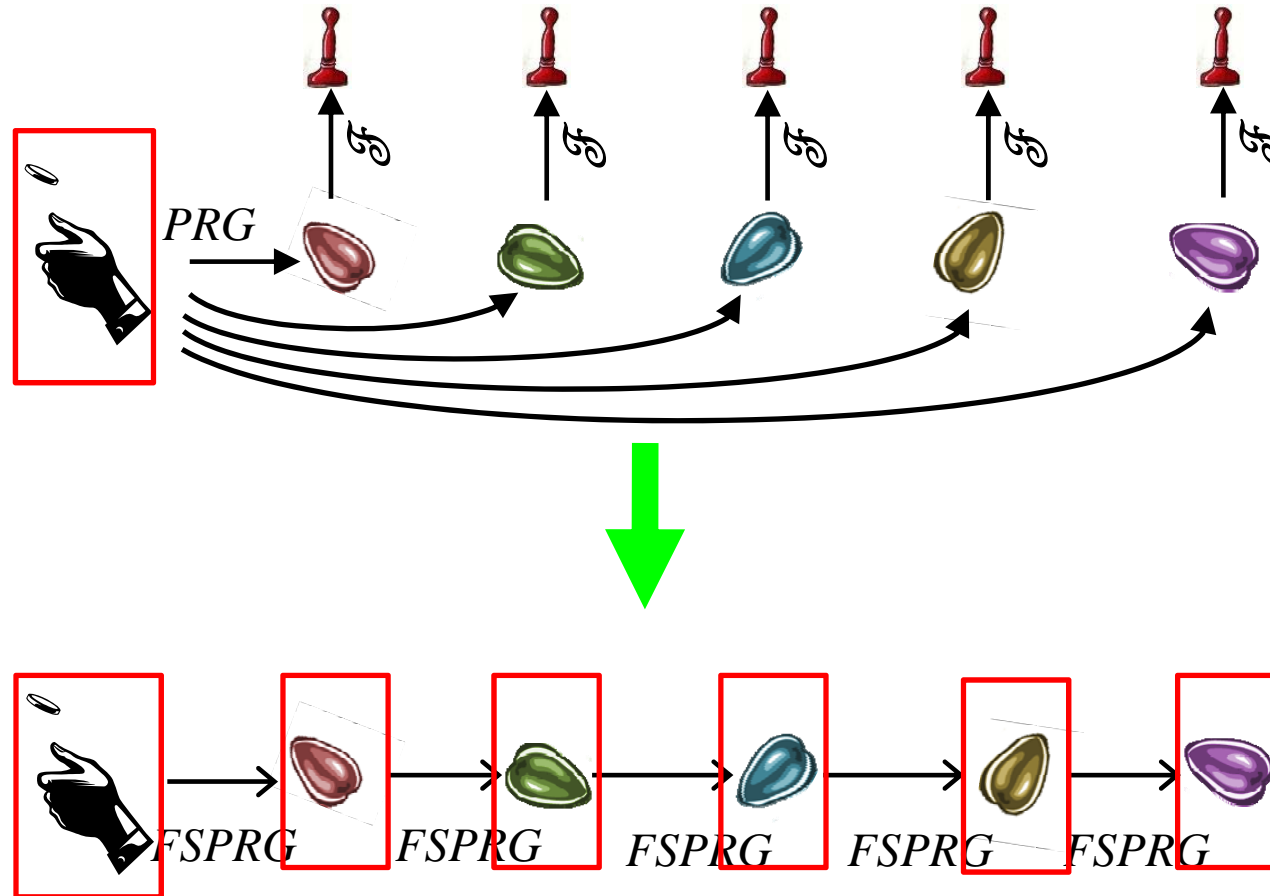
# Forward Secure Signatures



# XMSS forward secure



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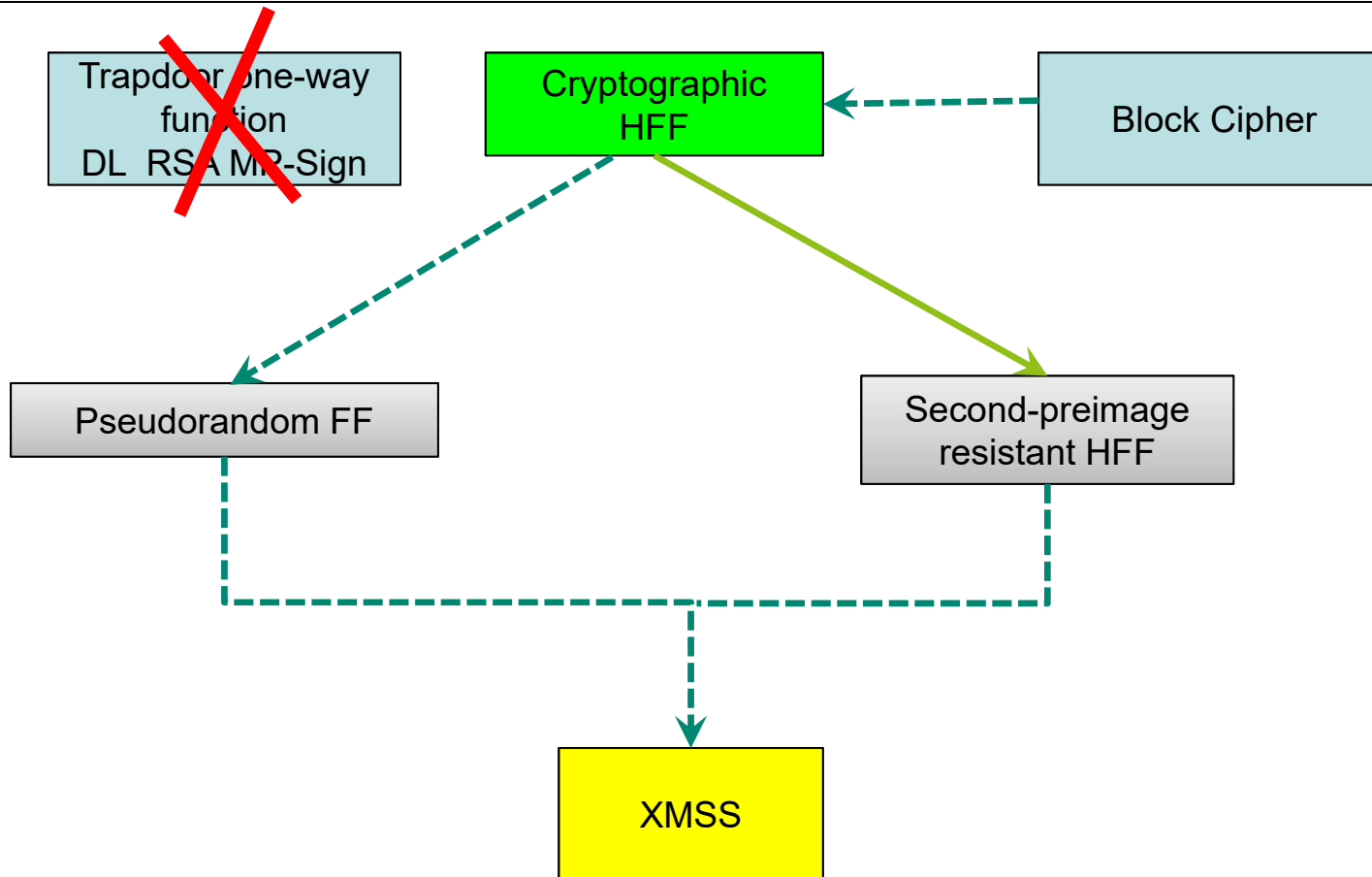




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# XMSS in practice

# XMSS in practice



# Hash functions & Blockciphers



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AES

Blowfish

3DES

Twofish

Threefish

Serpent

IDEA

RC5

RC6

...

SHA-2

SHA-3

BLAKE

Grøstl

JH

Keccak

Skein

VSH

MCH

MSCQ

SWIFFTX

RFSB

...

# XMSS performance (IETF-compliant version)



	Parameters				Cyclecounts [k-cycles]	Sizes [kB]		
	m	n	h	d	Signing	Signature	Secret key	Public key
XMSS	276	256	20	1	35 499	2.9	2.2	0.1
XMSS	316	256	60	3	44 882	8.8	14.6	0.1
RSA-15360	/	/	/	/	~ 2 256 000	1.9	1.9	1.9

256 bits classical security

- m: input length (bits)
- n: hash output length (bits)
- h: tree height
- d: # tree layers



# XMSS transfer project

Denis Butin, Stefan Gazdag



Standardisation underway at IETF/IRTF Crypto Forum Research Group

Internet-Draft *XMSS: Extended Hash-Based Signatures*:

<https://datatracker.ietf.org/doc/draft-irtf-cfrg-xmss-hash-based-signatures/>

Includes hierarchical (MT) variant

Shepherding successful, ready for IRTF chair review → RFC soon

<http://www.square-up.org/>

Practical Hash-based Signatures





## PQCrypto 2017

The Eighth International Conference on Post-Quantum Cryptography  
Utrecht, the Netherlands, June 26–28, 2017

